Antiderivatives

Ex An antiderivative of $f(x) = \cos(x)$ is $F(x) = \sin(x)$

Why?

$$F'(x) = \cos(x) = f(x)$$

Another?

$F(x) = \sin(x) + 3$

General Antideriv.

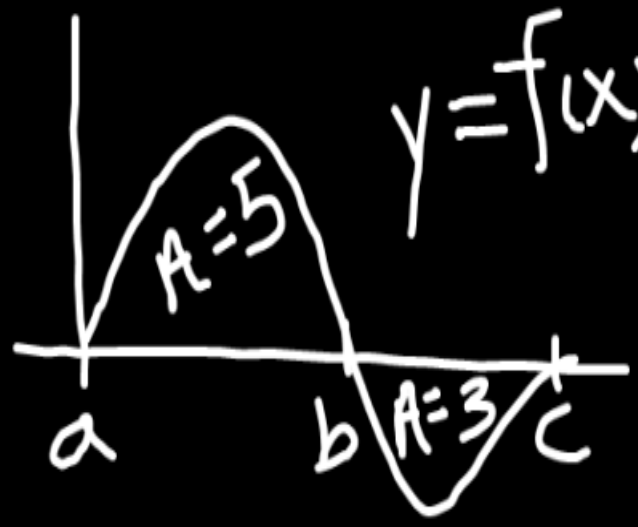
$F(x) = \sin(x) + C$

Integral Notation

If $F'(x) = f(x)$, then we write

$$\int f(x)\,dx = F(x) + C$$

Definite Integral



$$\int_b^c f(x)\,dx = -3$$

Then

$$\int_a^c f(x)\,dx = 5 + (-3)$$

$$= 2$$

FTOC

If $F'(x) = f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Ex:
$$\int_0^{\pi/2} \cos(x)\,dx = \sin(x)\Big|_0^{\pi/2} = \sin\left(\frac{\pi}{2}\right) - \sin(0)$$
$$= 1 - 0 = 1$$

Substitution

Recall Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

or

$$\int f'(g(x))\, g'(x)\, dx = f(g(x)) + C$$

Ex 1.

$$\int (x^2+4)^5 \, 2x \, dx = f(g(x)) + C$$

$g'(x)$!

$g(x) = x^2+4$

$f'(u) = u^5$

$f(u) = \frac{u^6}{6}$

$$= \frac{(x^2+4)^6}{6} + C$$

$\therefore$

Check

$$\frac{d}{dx}\frac{(x^2+4)^6}{6} = \frac{d}{dx}\frac{1}{6}(x^2+4)^6$$

$$= \frac{1}{6}6(x^2+4)^5 \cdot 2x$$

$$= (x^2+4)^5 2x$$ ✓

Note

u du

$$\int f'(g(x)) \cdot g'(x)\,dx = f(g(x)) + C$$

Let $u = g(x)$. Then $du = g'(x) \cdot dx$. Re write the integral in terms of u

$$\int f'(u)\,du = f(u) + C = f(g(x)) + C$$

Ex. 1 (take 2)

$$\int (x^2+4)^5 \, 2x\,dx$$

let $u = x^2+4$

then $du = 2x\,dx$

u^5 du

$$= \int u^5 \, du = \frac{u^6}{6} + C = \frac{(x^2+4)^6}{6} + C$$

✓

Ex 2

$$\int 4x^3\sqrt{x^4+16}\,dx = \int 4x^3\left(x^4+16\right)^{1/2}dx$$

du over $4x^3\,dx$; $u^{1/2}$ over $\left(x^4+16\right)^{1/2}$

let $u = x^4+16$

then $du = 4x^3\,dx$

$$= \int u^{1/2}\,du$$

$$= \frac{2}{3}u^{3/2}+C = \frac{2}{3}\left(x^4+16\right)^{\frac{3}{2}}+C$$

Ex 3

$$\int x^3 \sqrt{x^4+16}\,dx$$

Let $u = x^4+16$

then $du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\frac{1}{4}du$ ← $\sqrt{x^4+16}$ → $u^{1/2}$

$$= \frac{1}{4}\int u^{1/2}\,du$$

$$= \frac{1}{4}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \frac{1}{6}\left(x^4+16\right)^{3/2} + C$$