Probability | Homework 3 | November 6, 2013
Math 421 | | Dr. Pendergrass

ON MY HONOR, I HAVE NEITHER GIVEN NOR RECEIVED ANY AID ON THIS WORK, NOR AM I AWARE OF ANY BREACH OF THE HONOR CODE THAT I SHALL NOT IMMEDIATELY REPORT.

Pledged: ______________________________

Print Name: ______________________________

1. Suppose A and B are events in a probability space, with $P(A) = 0.3$ and $P(B) = 0.6$.
   (a) Given that A and B are independent, find the probability of $A \cup B$.
   (b) Given that $P(B|A) = 0.8$, find the probability of $A \cup B$.
   (c) Given that $P(A \cup B) = 0.8$, find the conditional probability of B given A.

2. Roll a pair of fair dice. What is the expected value of the product of the two dice?

3. Recall that a roulette wheel has 38 slots, 18 of which are red, 18 are black, and 2 are green. You have i dollars in your pocket, where i is an integer less than or equal to 100. You have resolved to bet one dollar on red repeatedly, until you either go bust, or you reach your goal of having \$100.
   (a) Let P_i be your probability of reaching your \$100 goal, given that you start with i dollars in your pocket, where $0 \leq i \leq 100$. Find a formula for P_i.
   (b) How large must your initial fortune be in order for you to have a better than even chance of reaching your \$100 goal?
   (c) Make an accurate plot of P_i as a function of i (you may use software).

4. Here is a game: there are three coins in an hat. Two are fair, while the third is two-headed. The player gets to choose a coin at random from the hat, and then flip it three times. Each time the flip results in heads, the player is paid \$1.
   (a) Let X be the player's winnings in this game. Find the probability mass function for X.
   (b) Suppose you want to charge people money to play this game. What is the minimum amount you'd need to charge in order to break even in the long run?

5. Let T have the geometric distribution with parameter p. Compute each of the following expected values:
   (a) $E\left[(3T-2)^2\right]$
   (b) $E\left[2^T\right]$
   (c) $E\left[e^{tT}\right]$, where t is an arbitrary real number.

6.
   (a) Toss a pair of fair dice repeatedly. Find the probability that a sum of 11 appears before a sum of 7 appears.
   (b) Let E and F be mutually exclusive events of some random experiment. Suppose that independent trials of this experiment are run. Find the probability that the event E occurs before the event F. (Your answer will of course depend on $P(E)$ and $P(F)$.)

7. Consider a Bernoulli trials process with success probability p. Let

$$C = \text{the first time two consecutive successes occur.}$$

   For example, if the sequence of successes and failures is $SFFSFSSF\cdots$, then $C = 7$. Clearly the possible values of C are $S_C = \{2, 3, 4, \cdots\}$. Find $P(C = 7)$.