On my honor, I have neither given nor received any aid on this work, nor am I aware of any breach of the Honor Code that I shall not immediately report.

Pledged: ______________________________

Print Name: ______________________________

1. Let X have the *discrete uniform distribution* on $\{1, 2, \cdots, n\}$, i.e. $P(X = k) = 1/n$ for $k = 1, 2, \cdots n$. Find the mean and variance of X.

2. Let X be a discrete random variable with probability mass function $p(x)$. The *entropy* of X is the quantity $H(X)$ defined by
$$H(X) = -E\left[\log_2(p(X))\right]$$
Like variance, entropy can be considered as a measure of uncertainty in a distribution. (In fact the units of entropy are *bits*.) Find the entropy of the Bernoulli(p) random variable. Plot $H(X)$ as a function of p, and comment.

3. Let X have possible values $\{1, 2\}$, let Y have possible values $\{1, 2, 3\}$, and suppose the joint mass function for (X, Y) is $p(x, y) = C(x + y)$ where $x \in \{1, 2\}$, $y \in \{1, 2, 3\}$, and C is a constant.
   (a) Find the value of C.
   (b) Find the marginal mass functions for X and Y.
   (c) Are X and Y independent? Explain.

4. Deal a five-card hand from well-shuffled deck, and let X be the number of hearts in the hand, and Y be the number of clubs. Find the joint probability mass function for X and Y.

5. Consider a Bernoulli trials process with success probability p. Let X_n be the number of successes up to time n. Find the joint probability mass function for X_m and X_{m+n}. (Hint: Find $P(X_m = i, X_{m+n} = i + j)$ using the multiplication principle for conditional probabilities.)

6. Suppose that X and Y are independent discrete random variables. Show that $E[XY] = E[X]E[Y]$.

7. The probability generating function for an integer-valued random variable X is
$$P(z) = \frac{1}{6} + \frac{1}{3}z + \frac{1}{2}z^2$$
   (a) Find the probability mass function for X.
   (b) Find the moment-generating function for X.
   (c) Suppose Y and Z are random variables with the same probability generating function as X, and that $\{X, Y, Z\}$ is independent. Find the probability mass function for $S = X + Y + Z$.

8. The probability generating function for a certain integer-valued random variable X is $P(z) = e^{\lambda(t-1)}$, where λ is a positive constant.
   (a) Find the probability mass function for X.
   (b) Find the mean and variance of X.