1. Recall that $\Gamma(1/2) = \sqrt{\pi}$, where $\Gamma$ is the gamma function. Find a formula for $\Gamma\left(\frac{2n+1}{2}\right)$ for $n = 1, 2, 3, \ldots$, and prove that your formula holds for all $n \in \mathbb{N}$.

2. My lamp uses two light bulbs. Luckily, I’ve got 2 bulbs, of identical make. Suppose that the lifetimes of my bulbs are IID exponential random variables with mean 1200 hours.
   (a) How long does it take for the first of the two bulbs to burn out, on average?
   (b) How long on average does it take for the remaining bulb to burn out?
   (c) Suppose instead of burning both bulbs simultaneously, I use them one at a time, replacing the first as soon as it burns out with the second. How long on average can I use the lamp?

3. Let $X$ have the gamma distribution with shape parameter $\alpha$ and scale parameter $\theta$. Let $Y = cX$, where $c$ is a positive constant. Show that $Y$ has the gamma distribution with shape parameter $\alpha$ and scale parameter $c\theta$.

4. Let $X \sim \Gamma(\alpha, \theta)$, where $\alpha > 1$, and let $Y = 1/X$.
   (a) Show that $E[Y] = \frac{1}{\theta(\alpha - 1)}$.
   (b) Find the PDF of $Y$. 