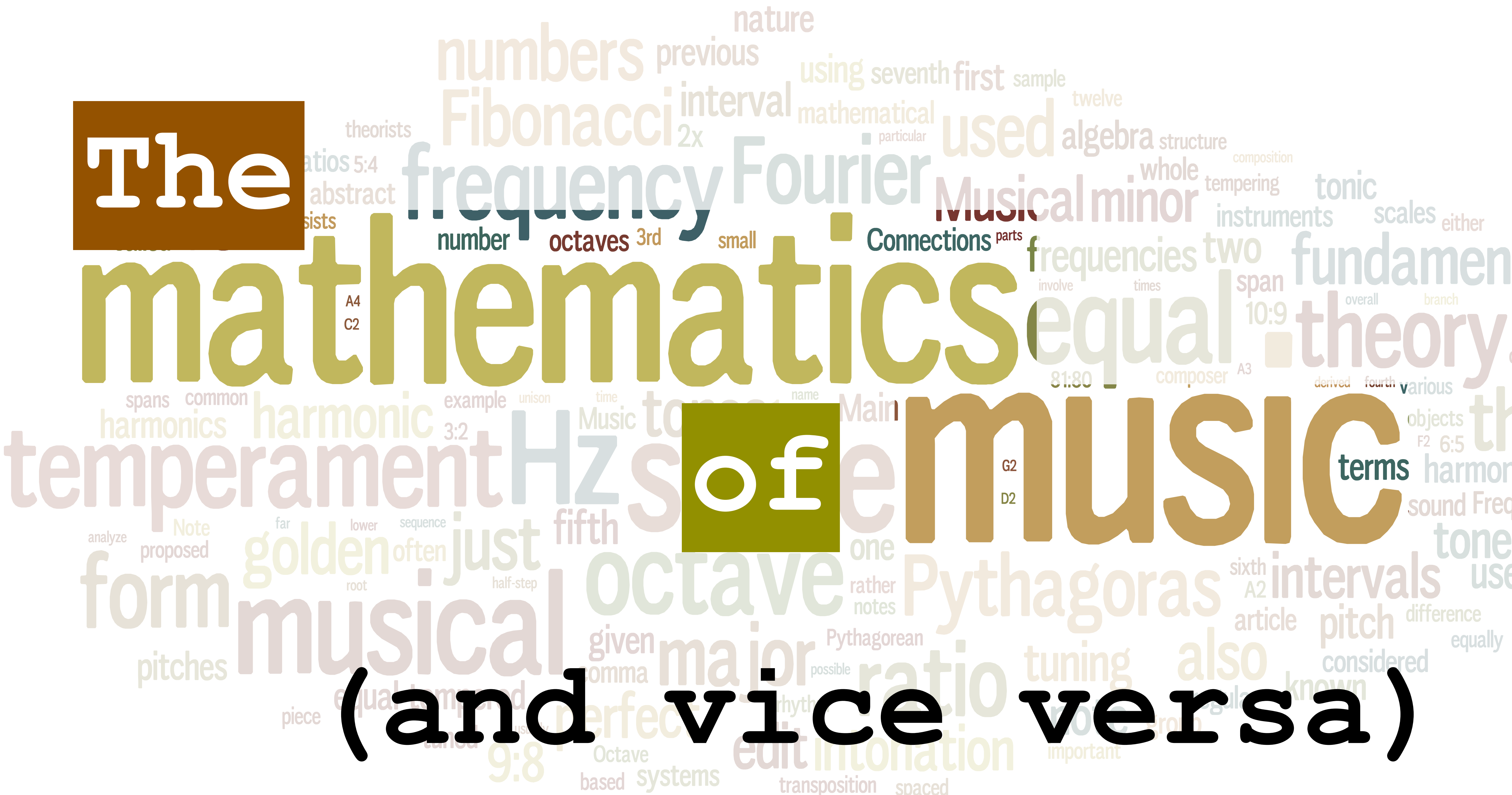
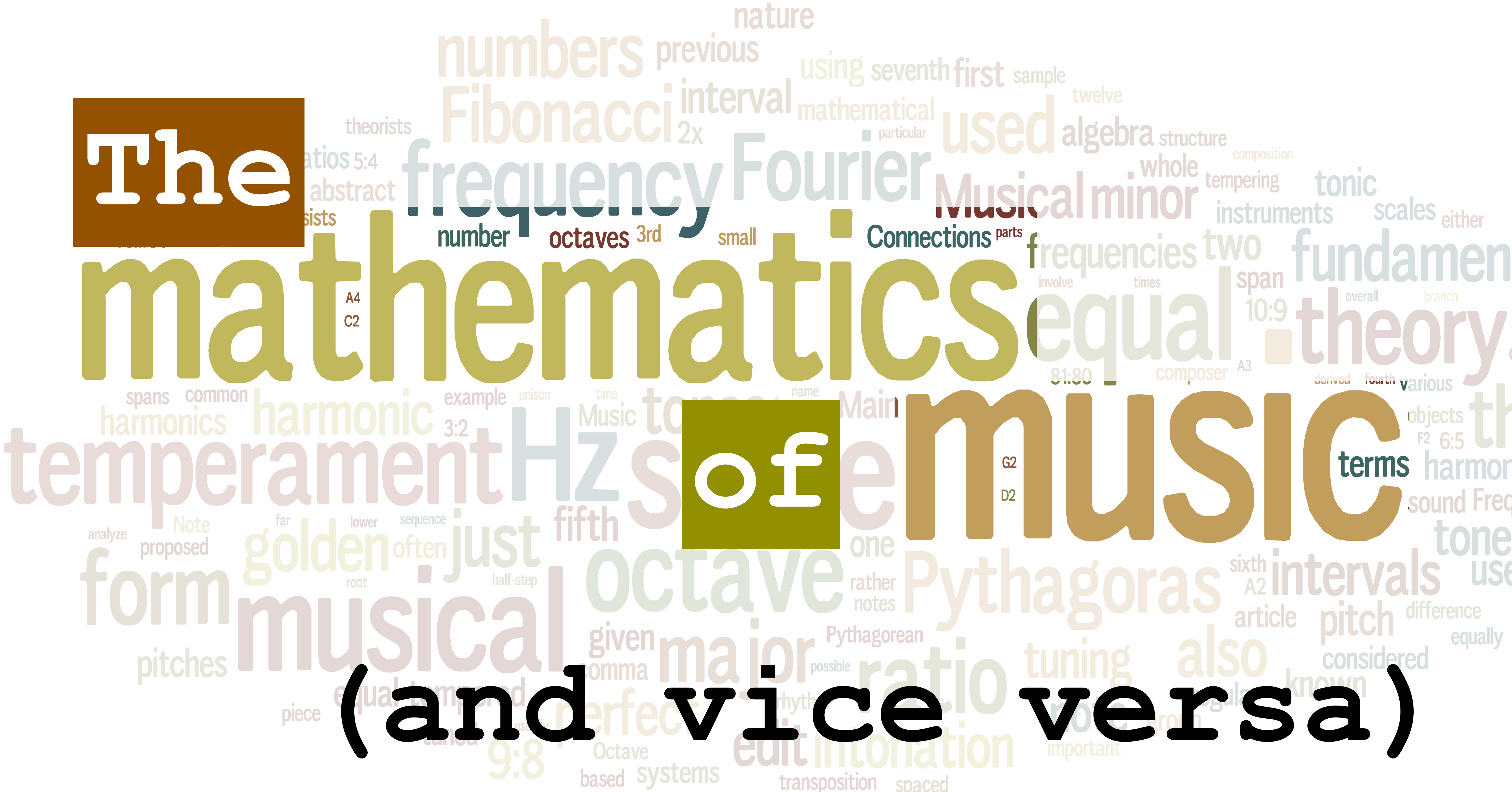
The mathematics of music (and vice versa)



Marcus Pendergrass, Hampden-Sydney College

Applied Mathematics, Fall 2012

The mathematics of music (and vice versa)



Marcus Pendergrass, Hampden-Sydney College

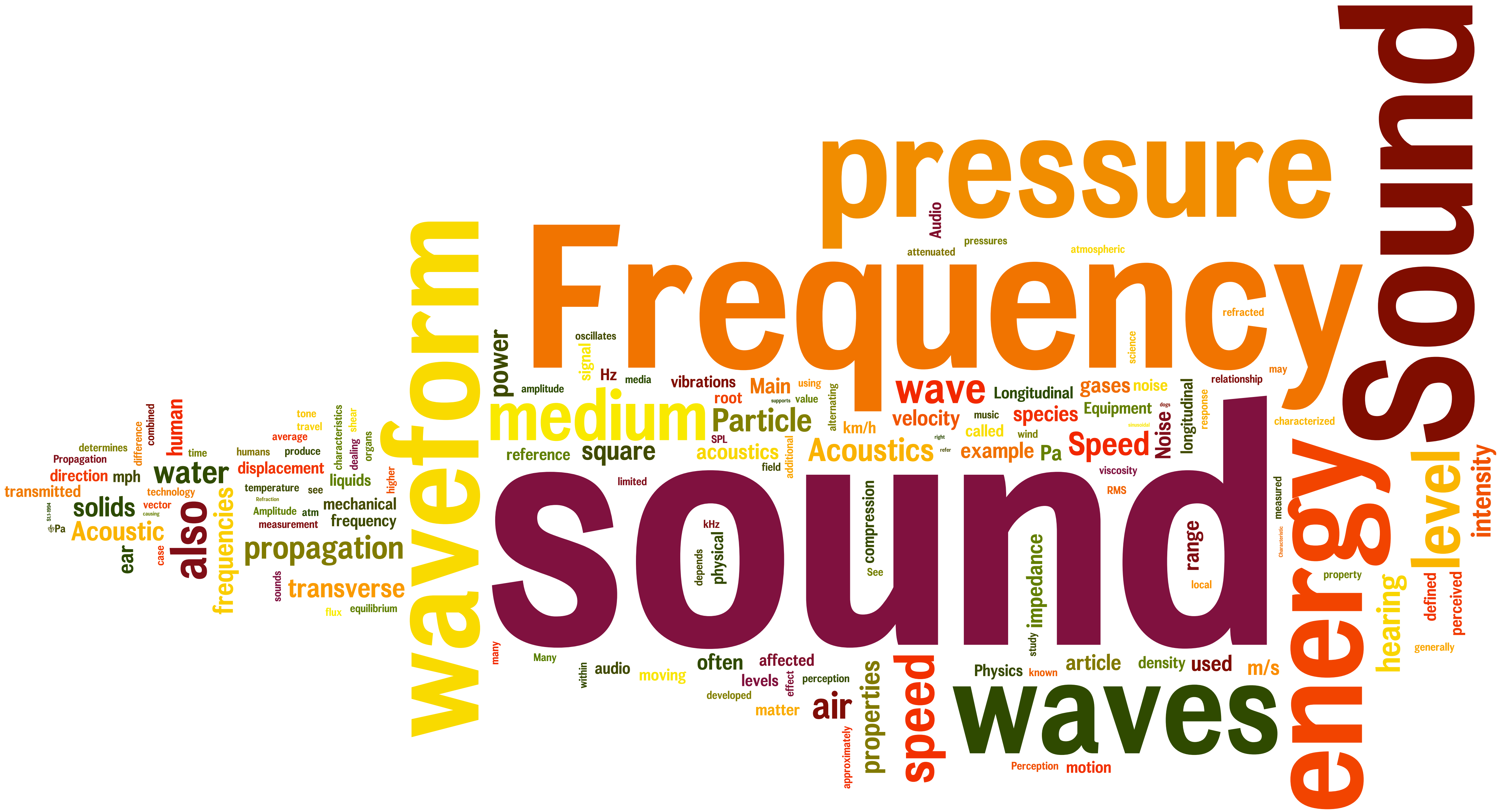
Applied Mathematics, Fall 2012

The mathematics of music (and vice versa)

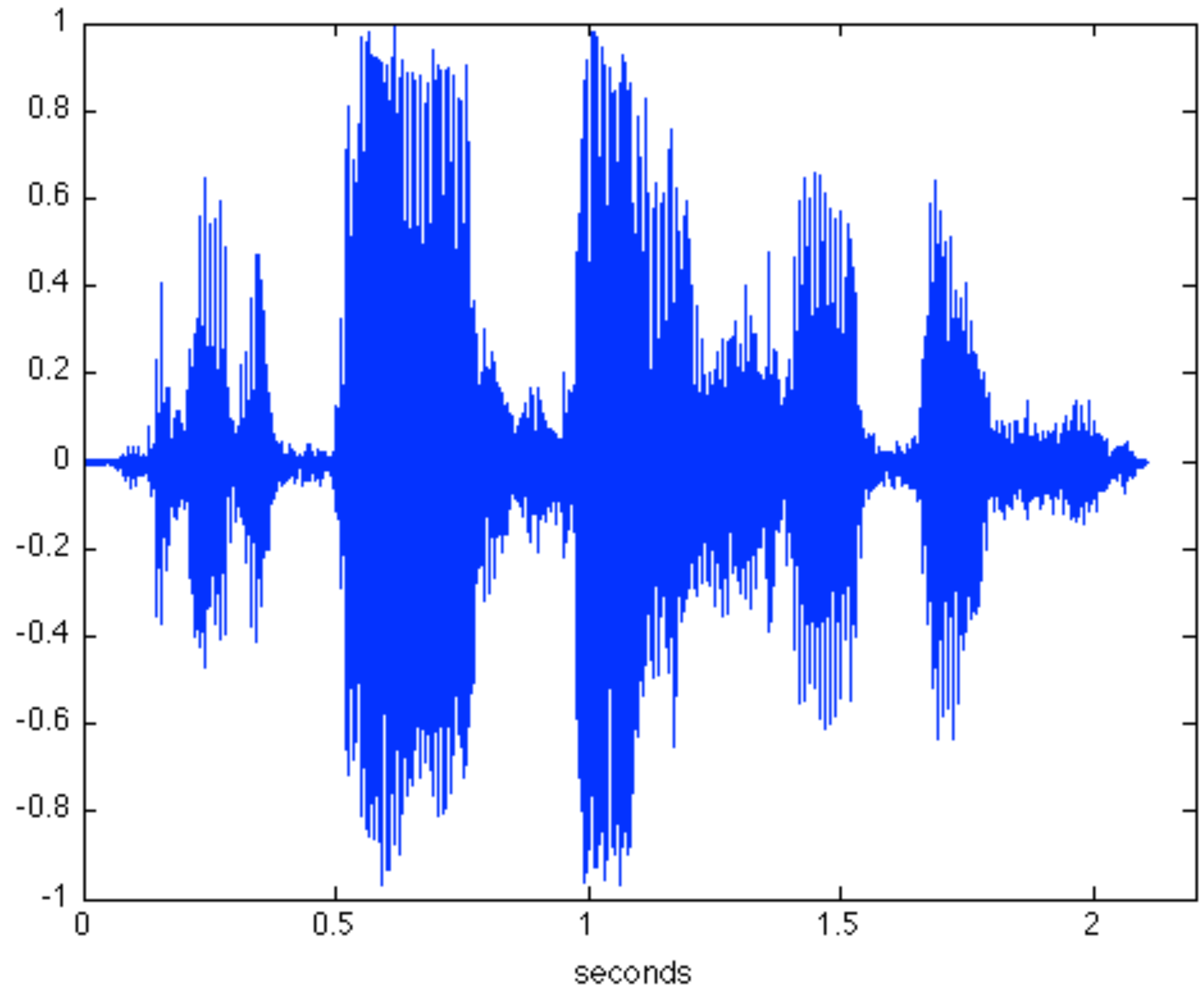
Marcus Pendergrass, Hampden-Sydney College

Applied Mathematics, Fall 2012

I. Some Sound Mathematics

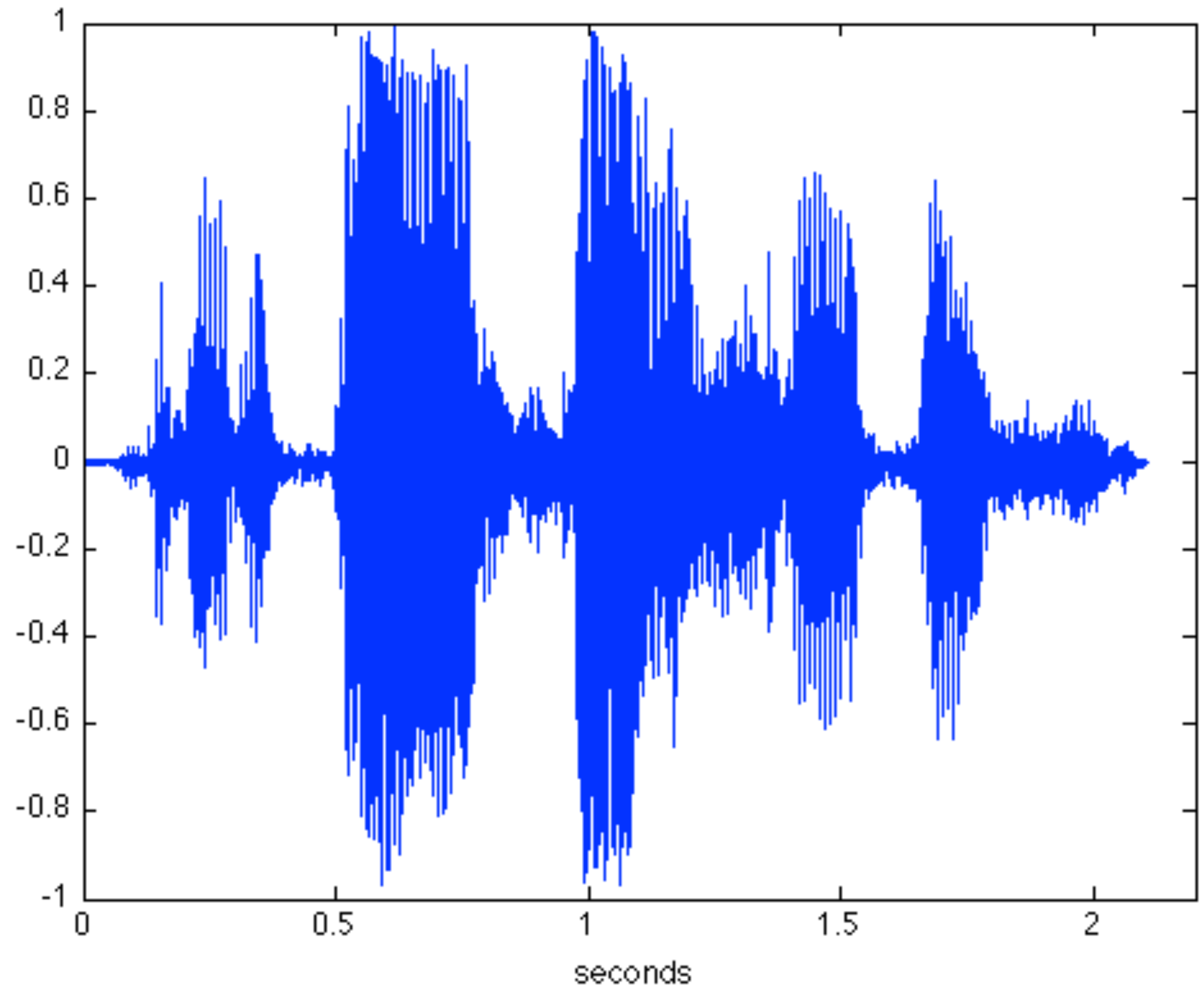


Sounds as waveforms



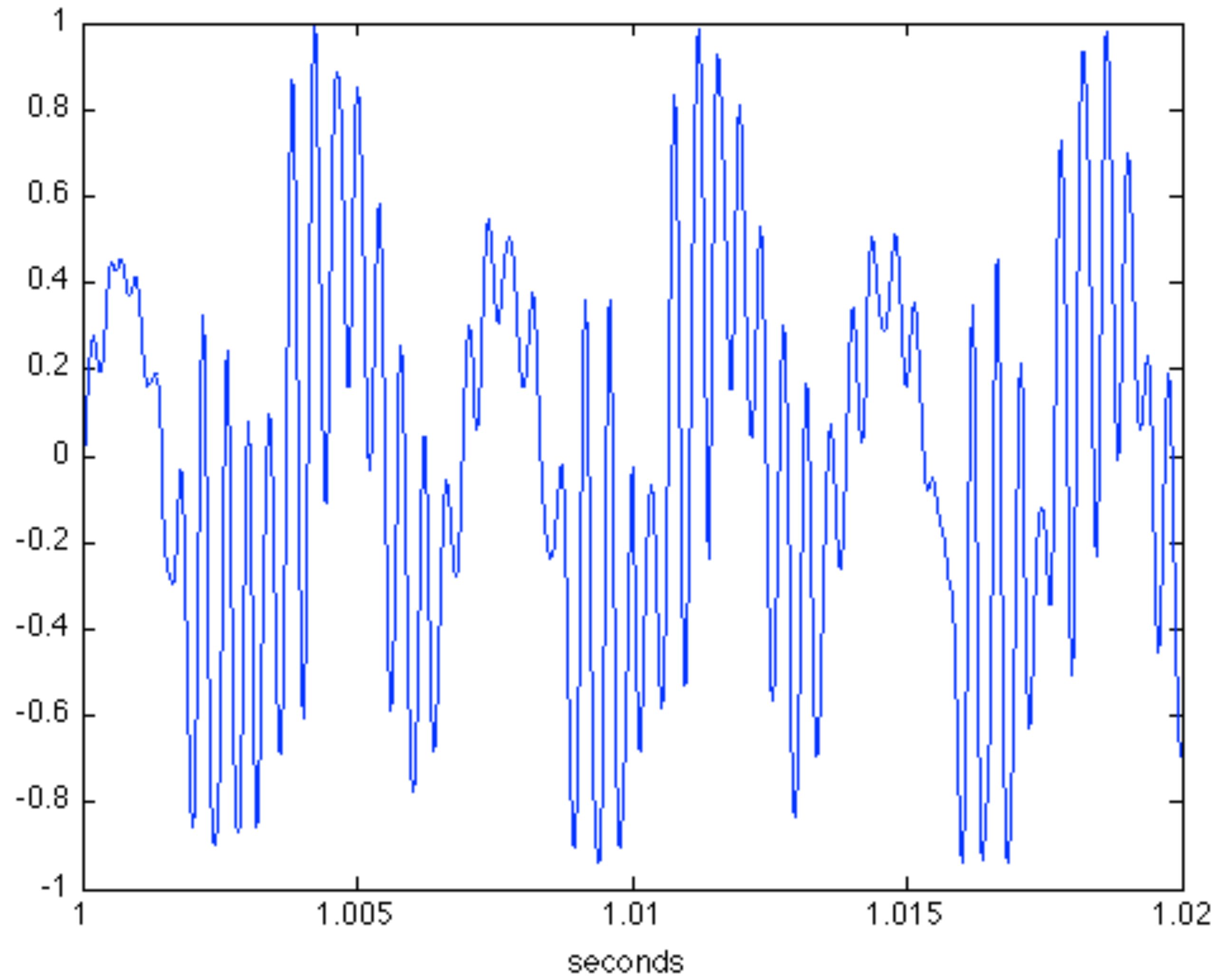
Where did this sound come from?

Sounds as waveforms



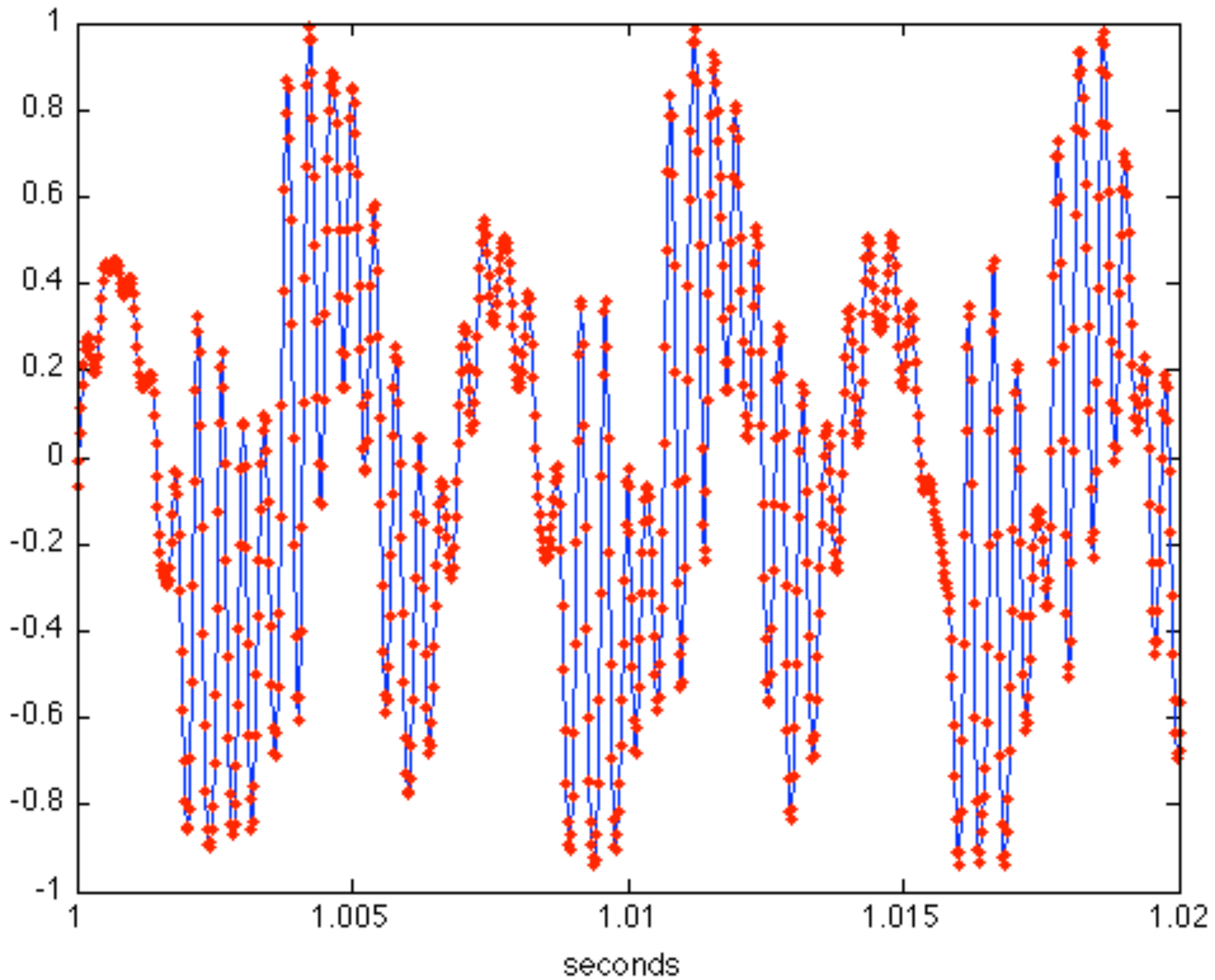
Where did this sound come from?

Sounds as waveforms



"I've got a bad **fe**eling about this..."

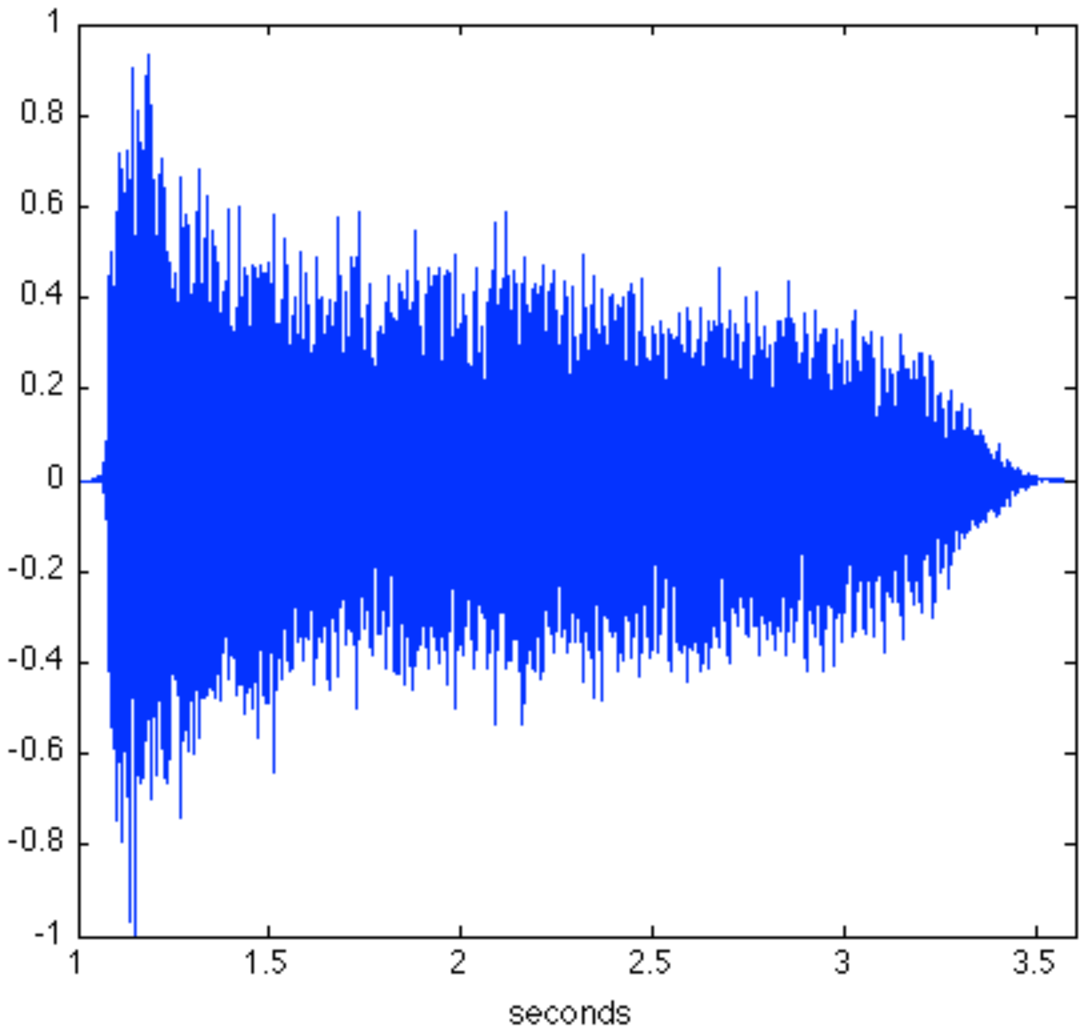
Sounds as waveforms

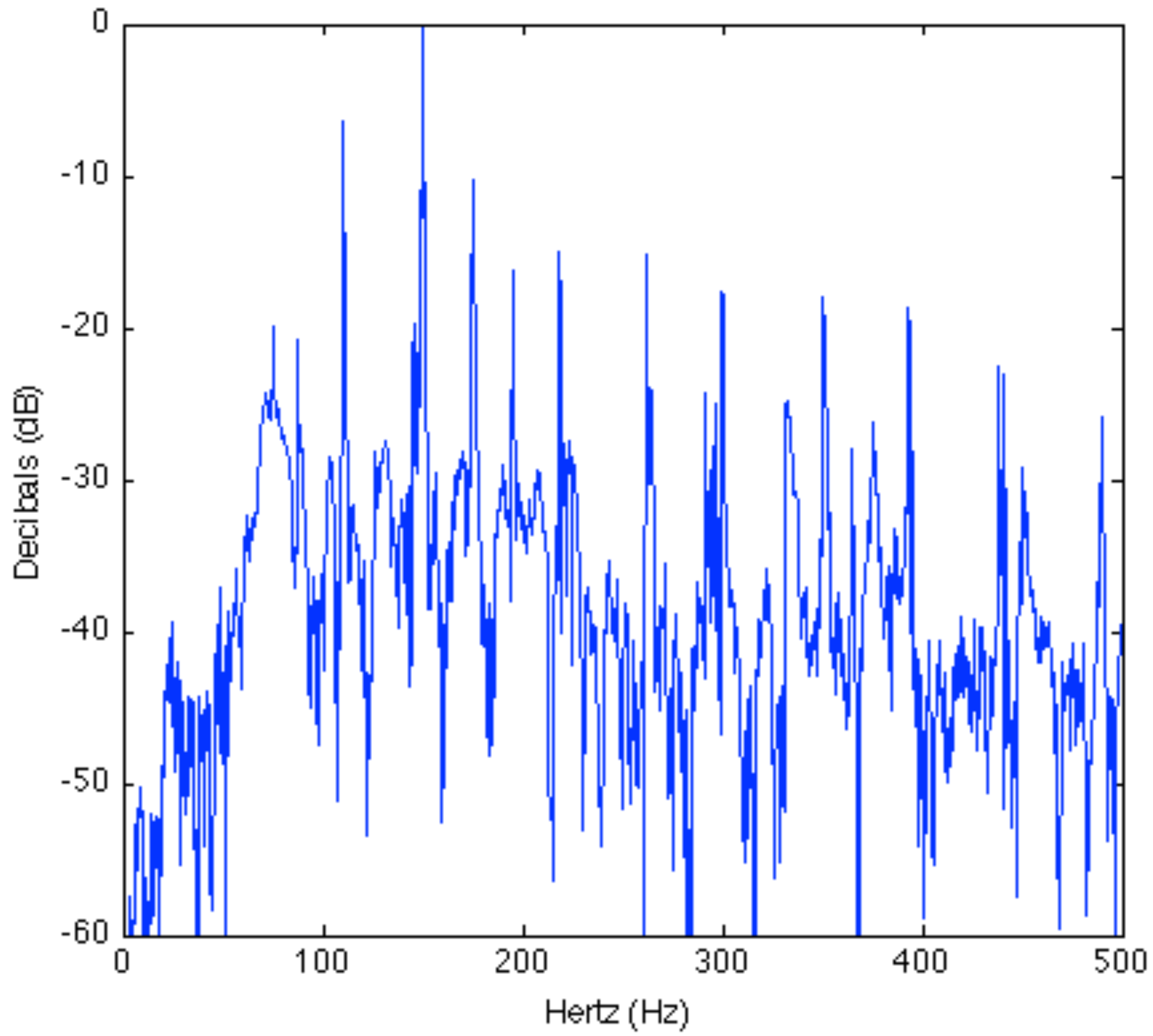


"I've got a bad **fe**eling about this..."

Sounds as waveforms

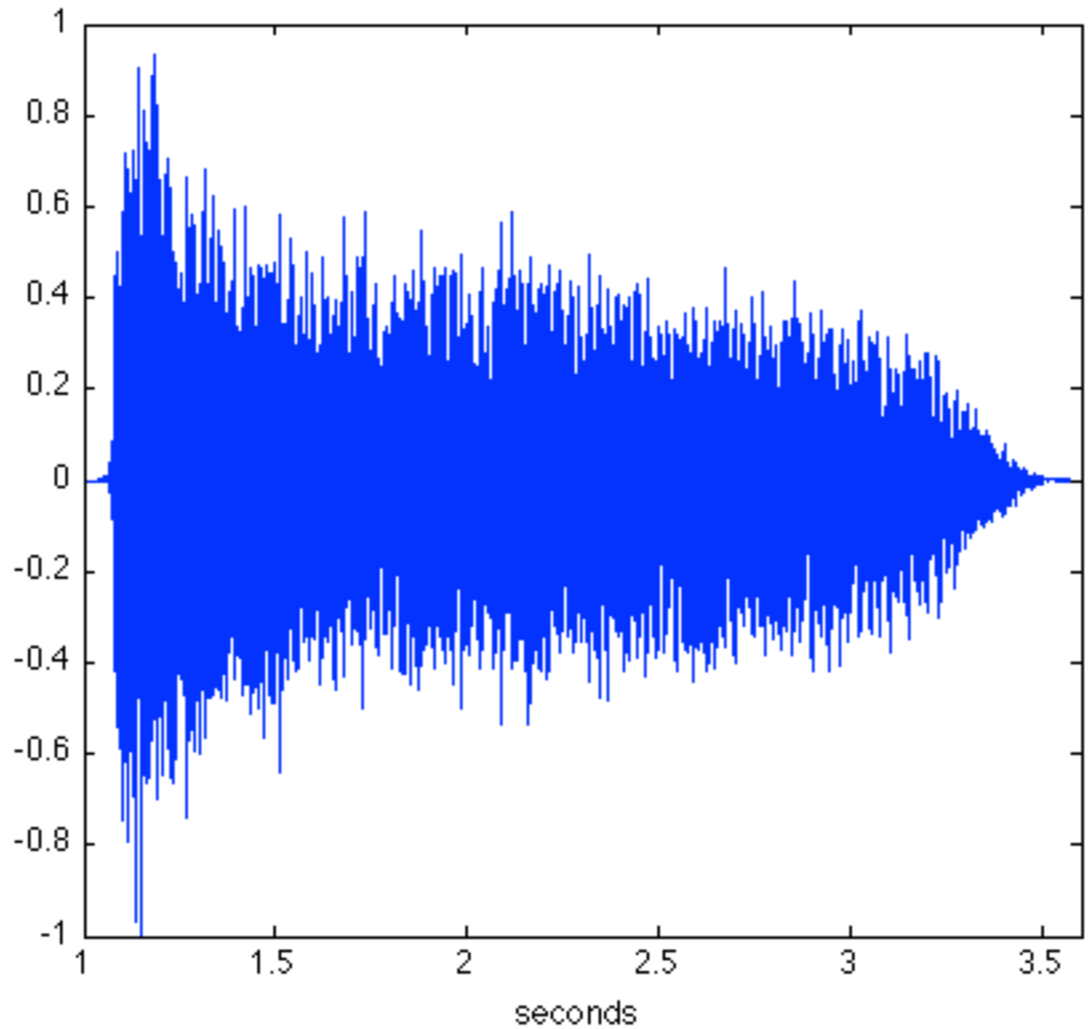
Frequency

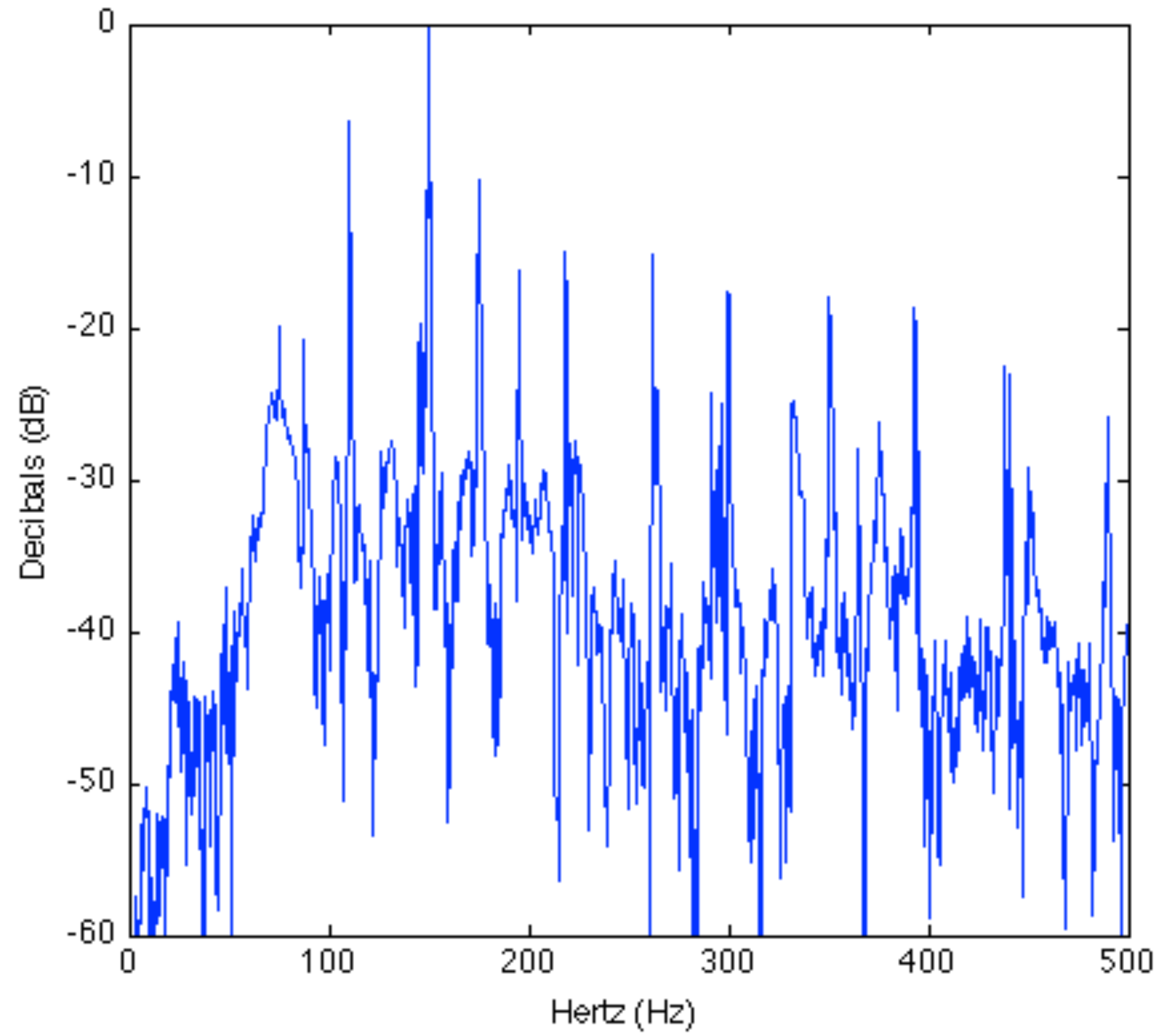




Recognize this?

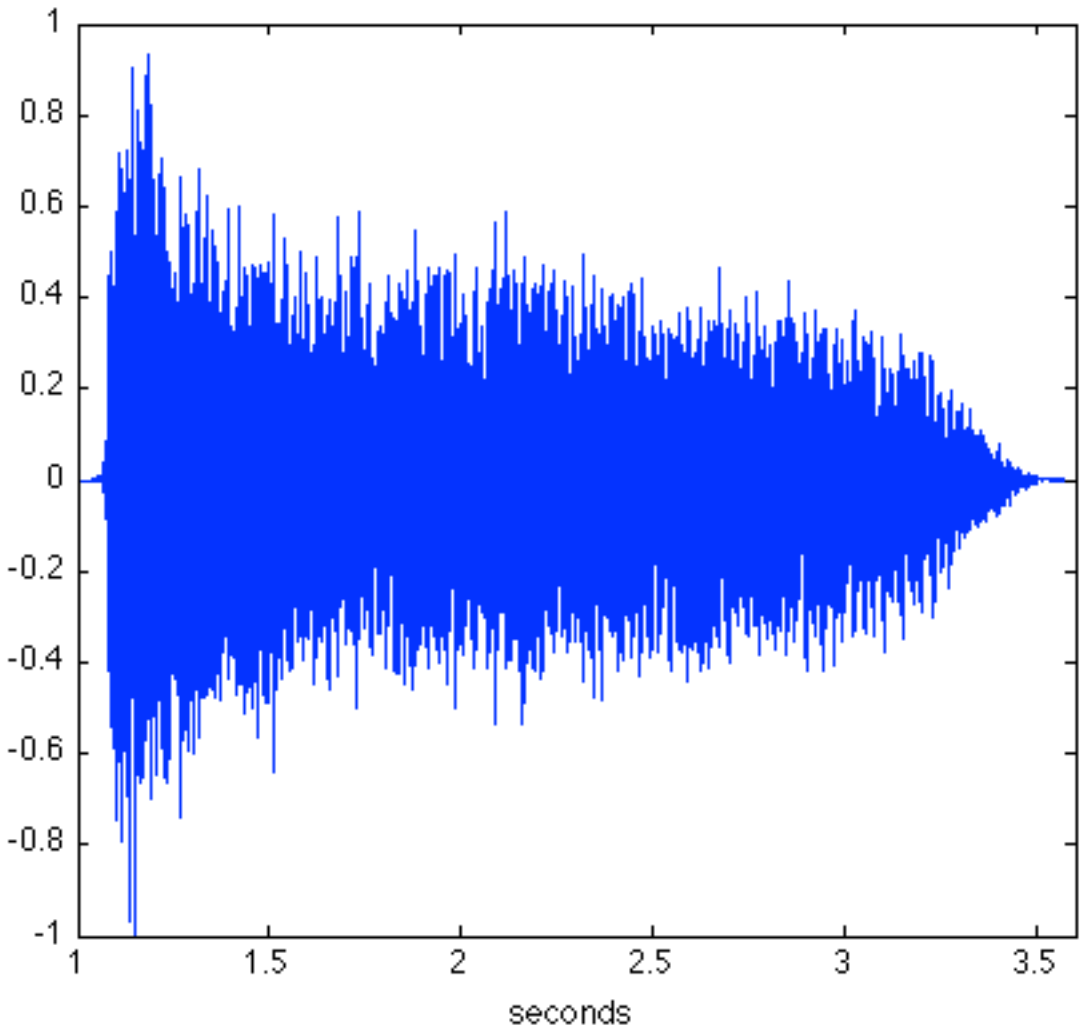
Frequency

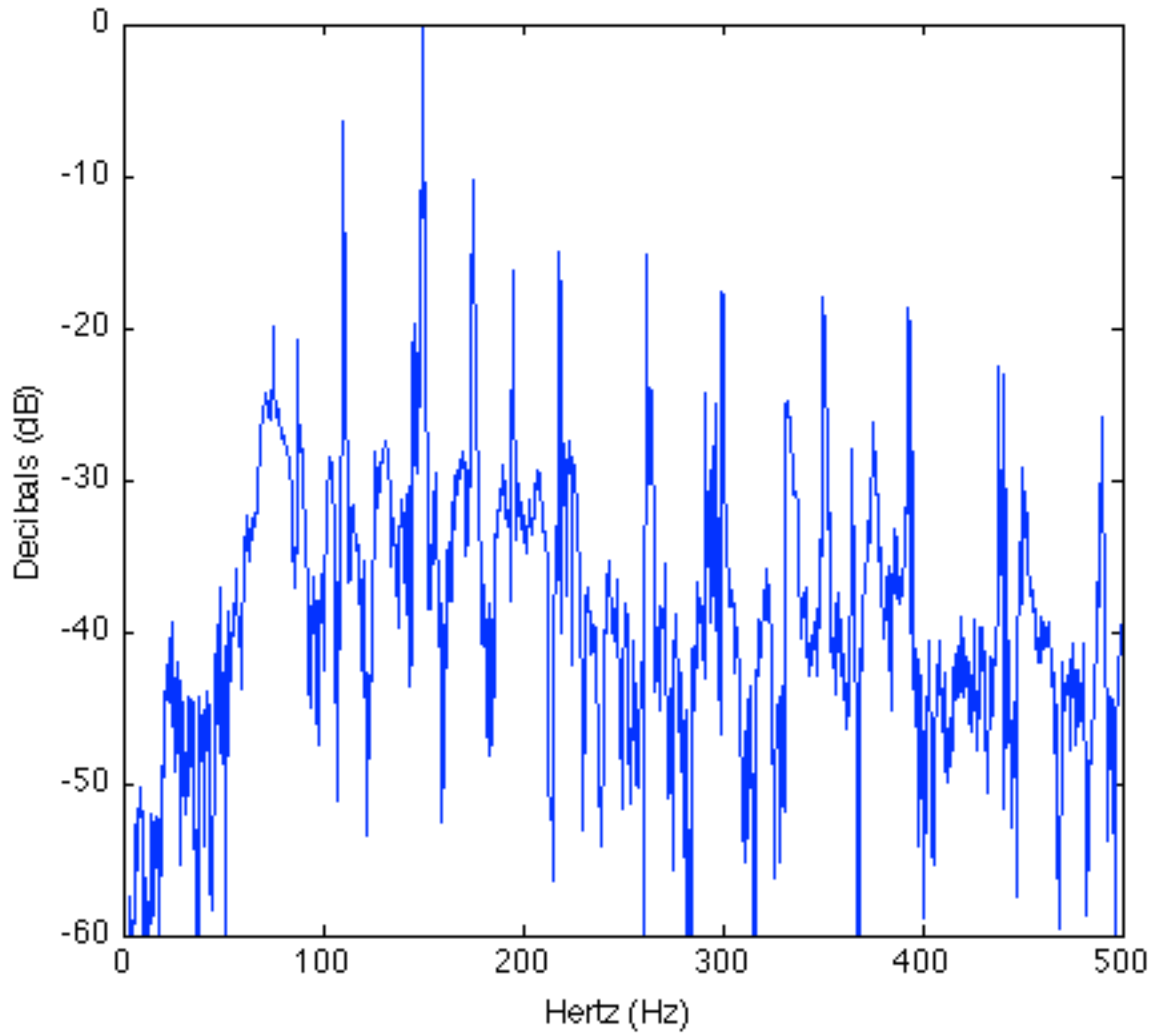




Recognize this?

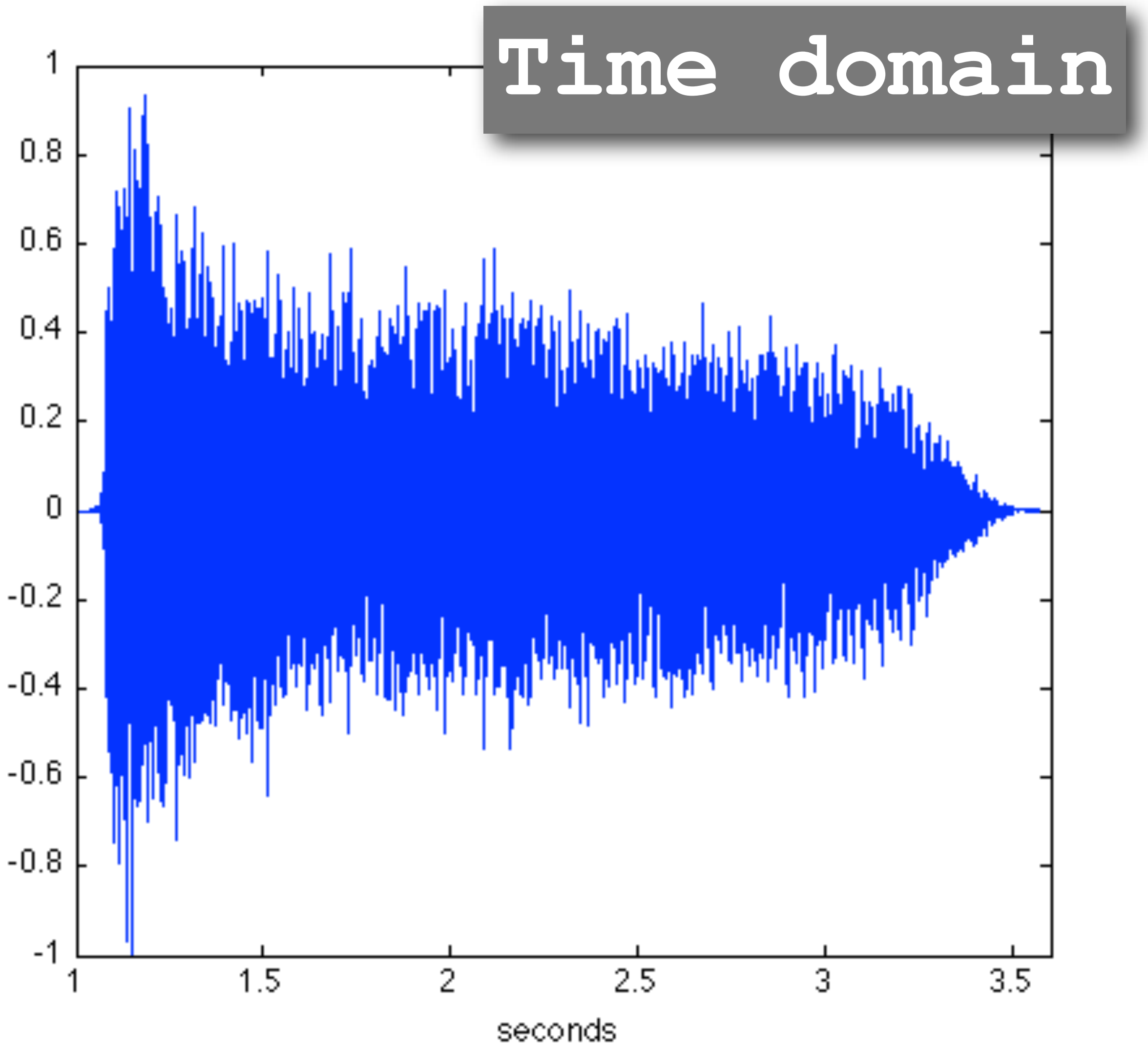
Frequency

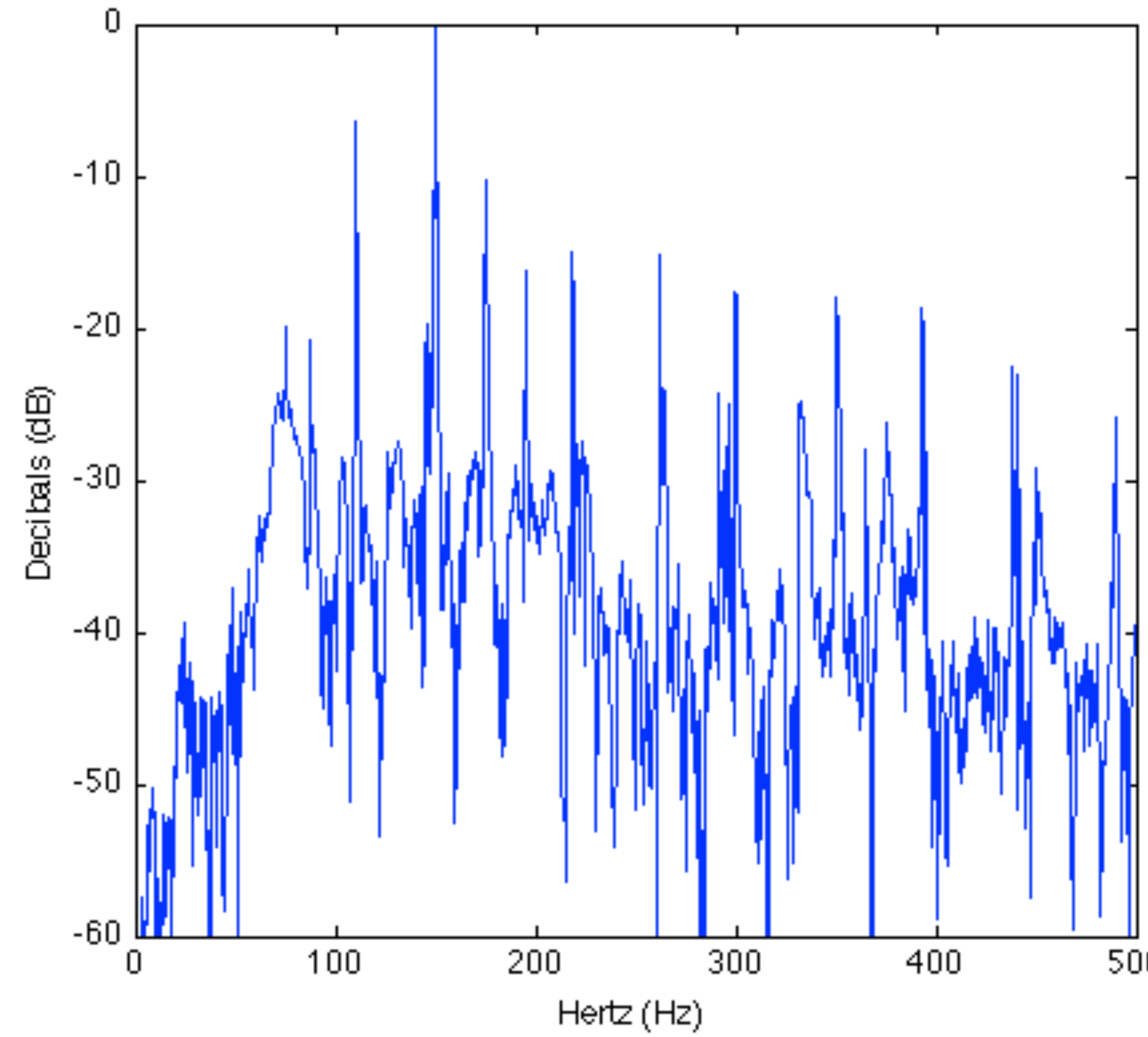




Recognize this?

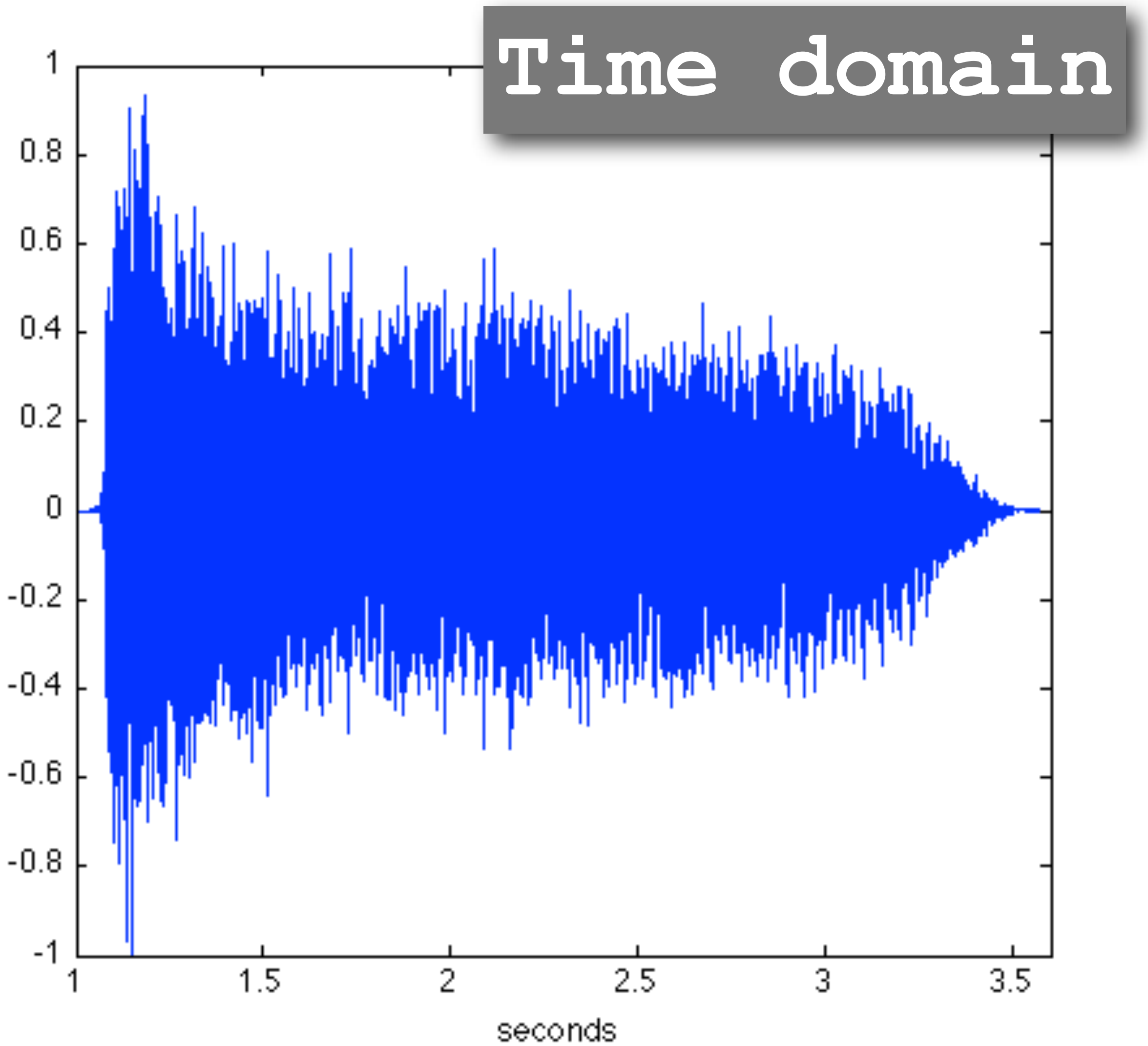
Frequency

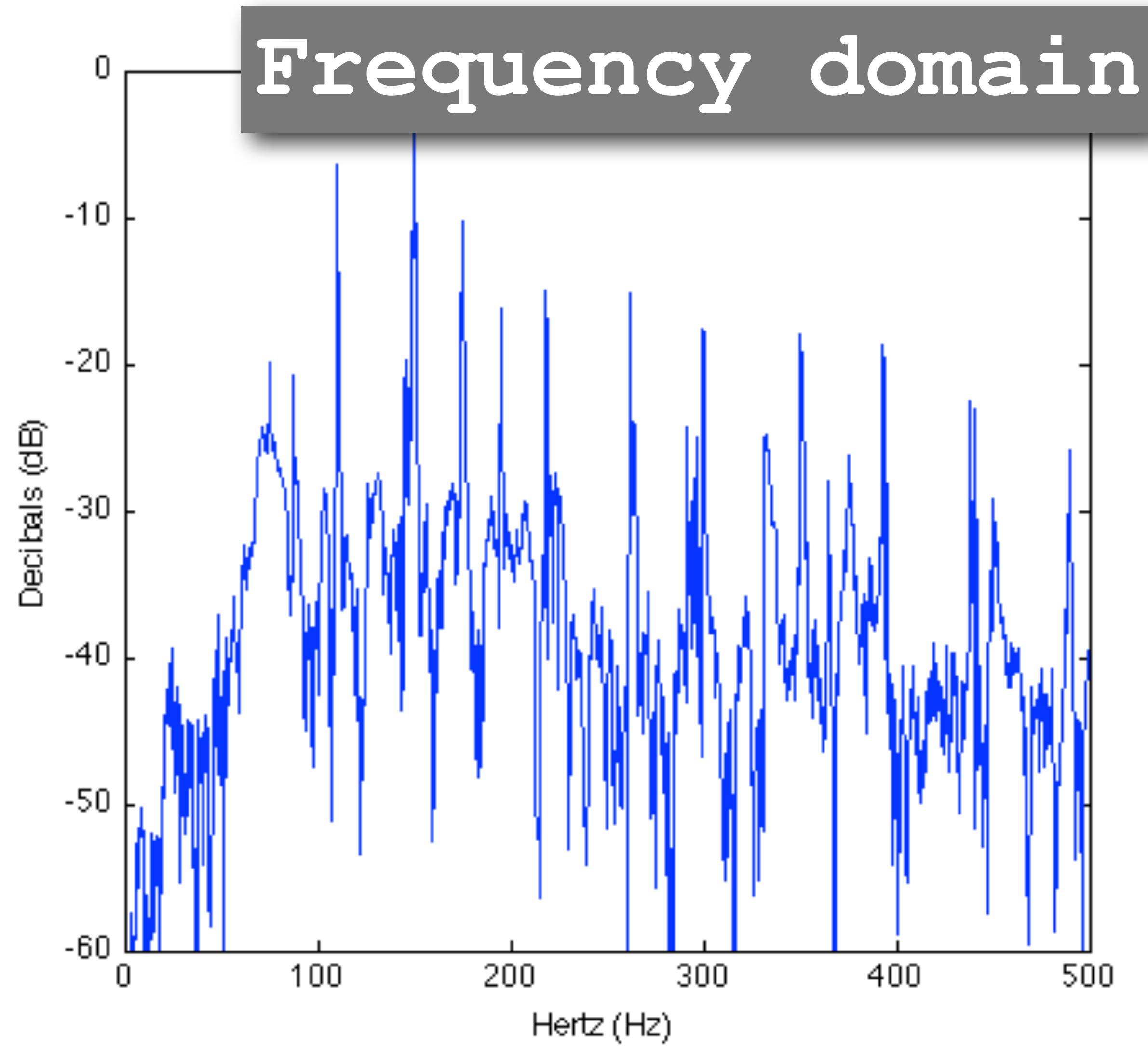




Recognize this?

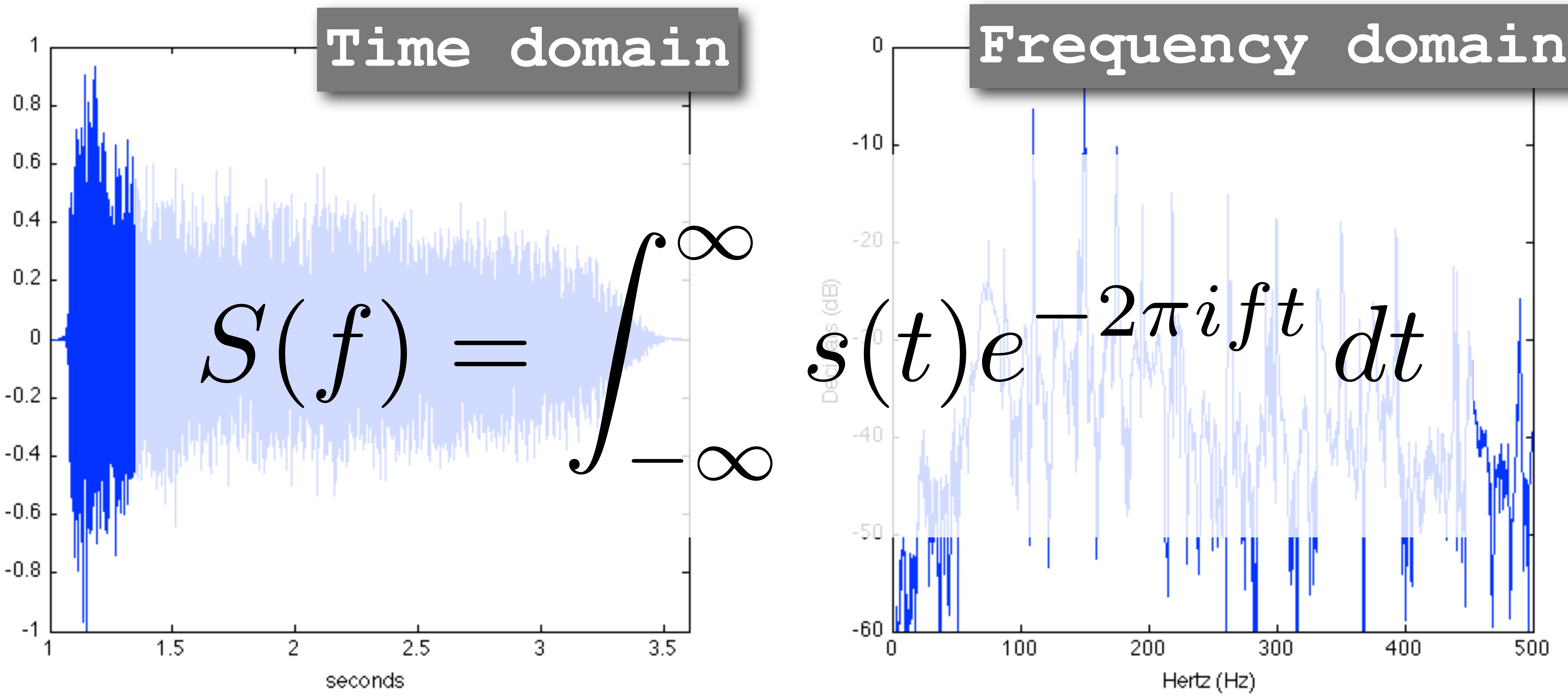
Frequency





Recognize this?

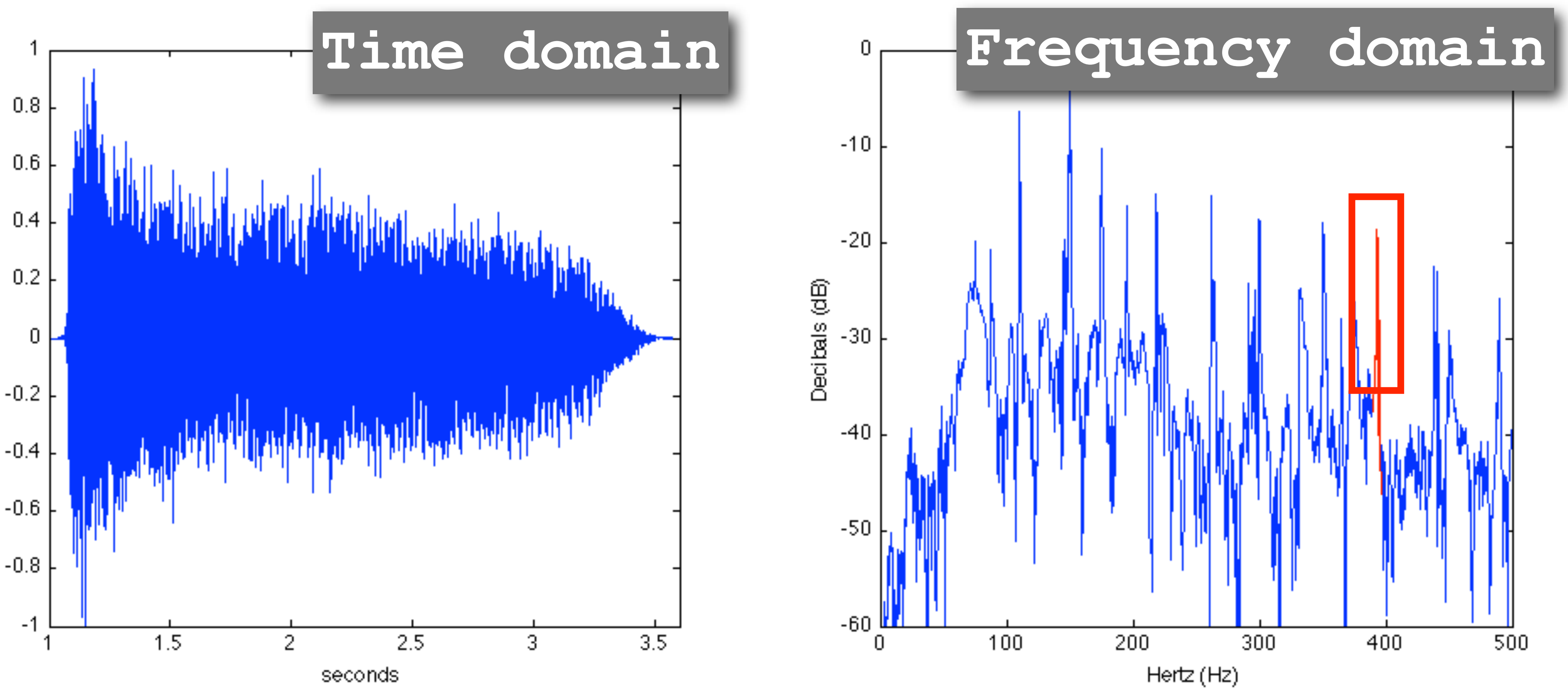
Frequency



$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-2\pi i f t} \, dt$$

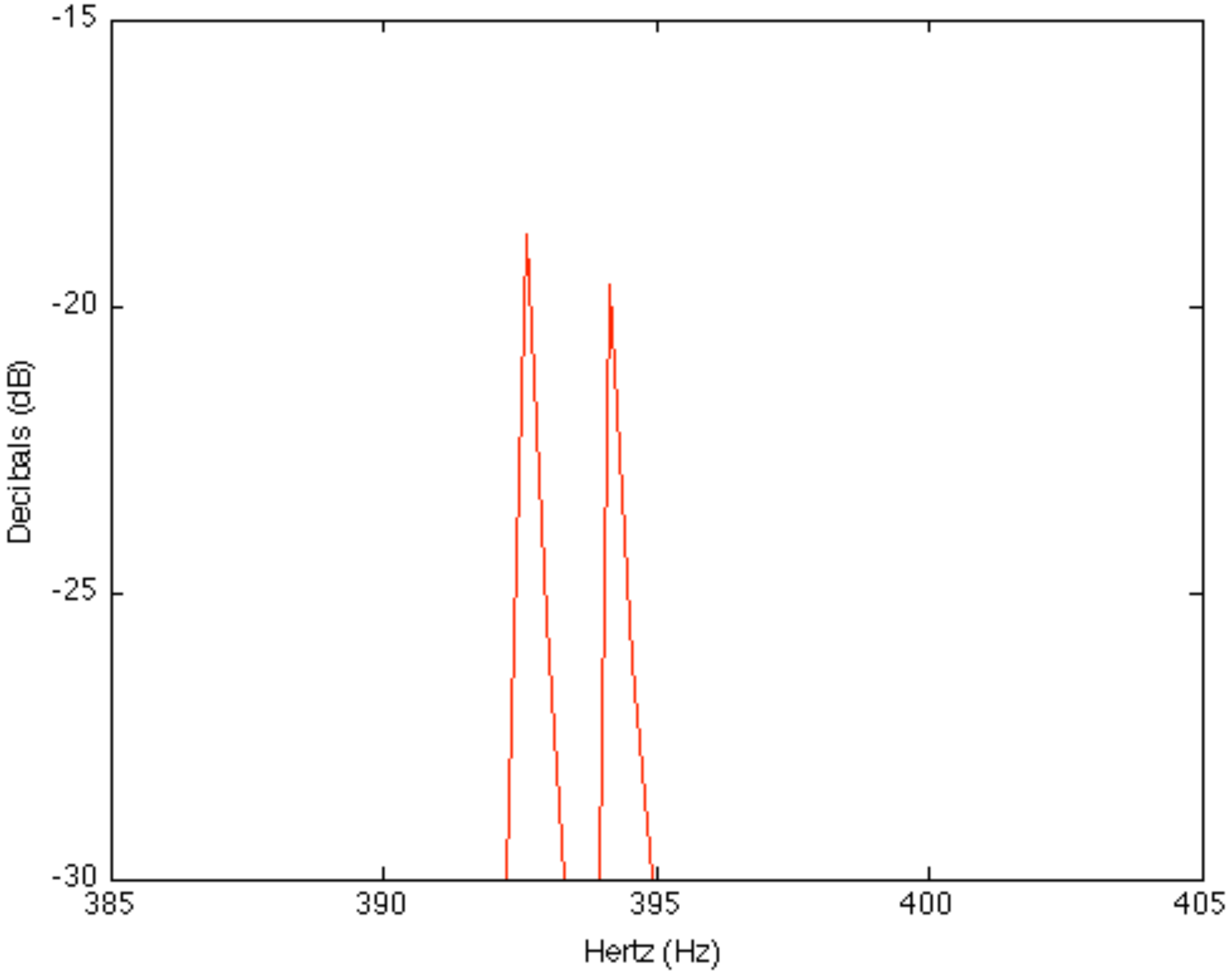
Recognize this?

Frequency

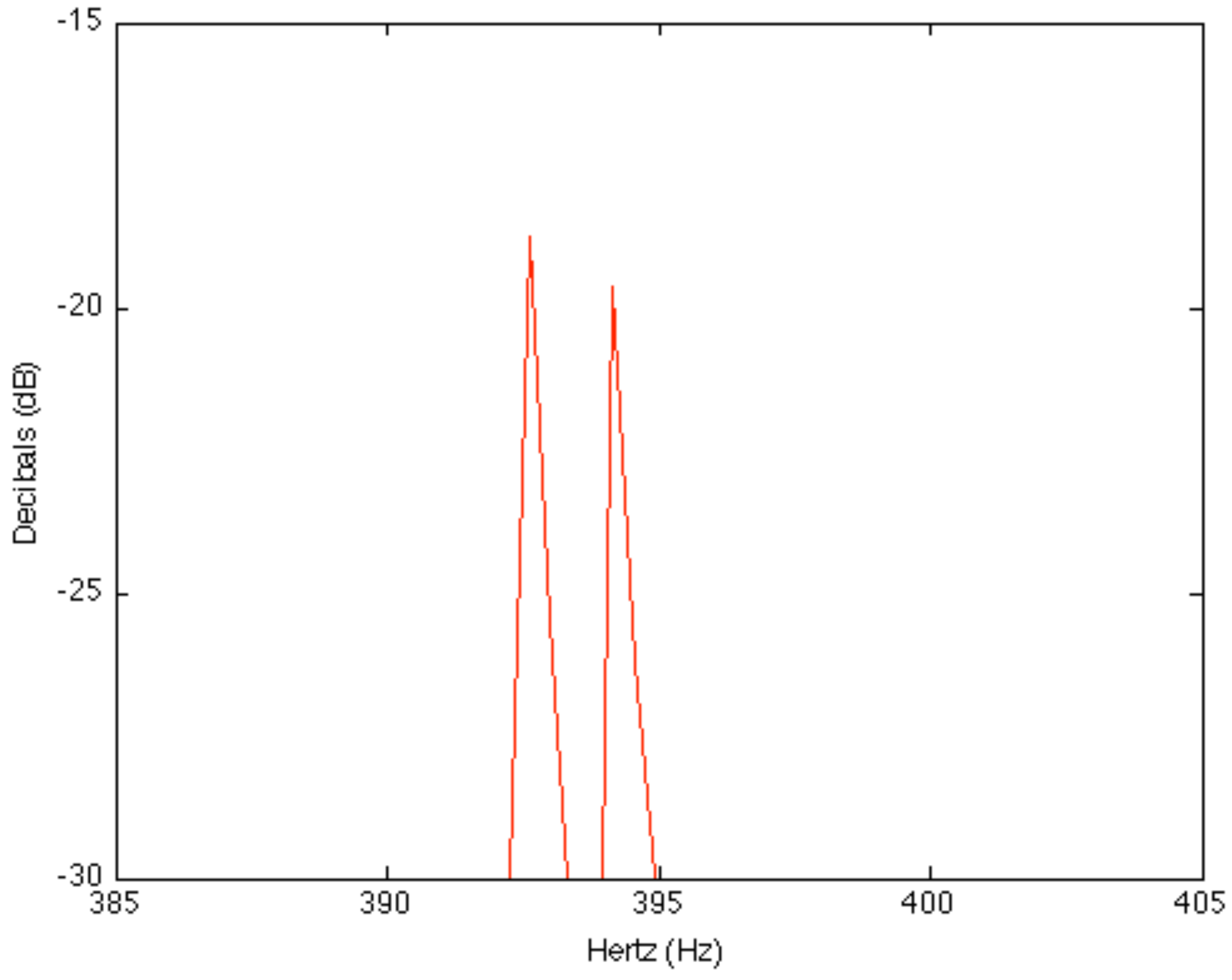


Recognize this?

Frequency

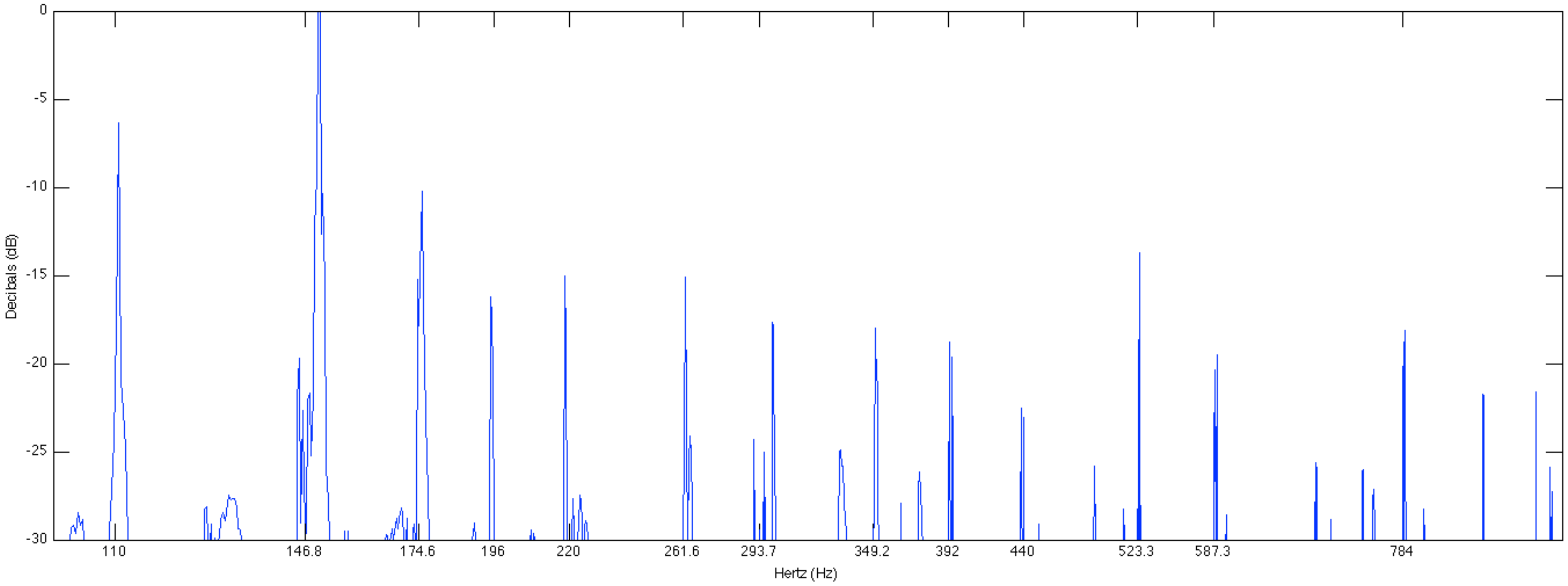


Frequency

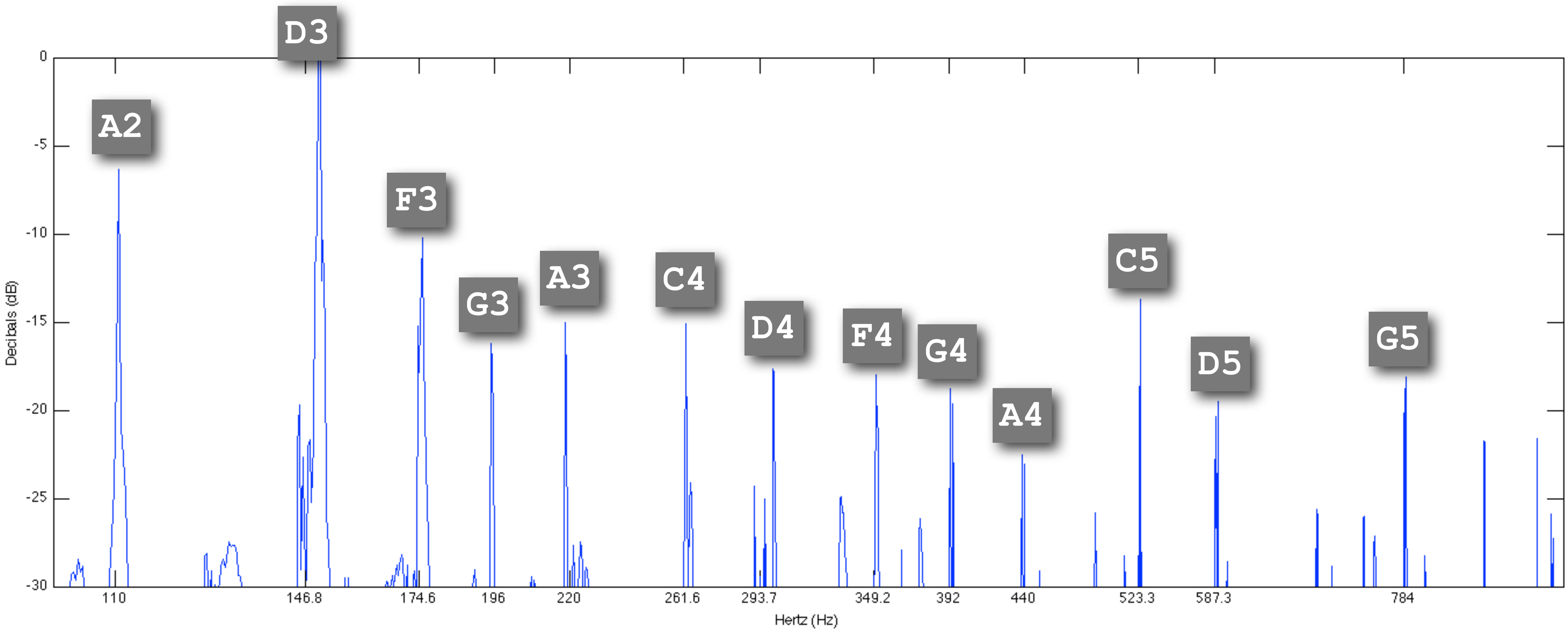


George played a 12-string guitar!

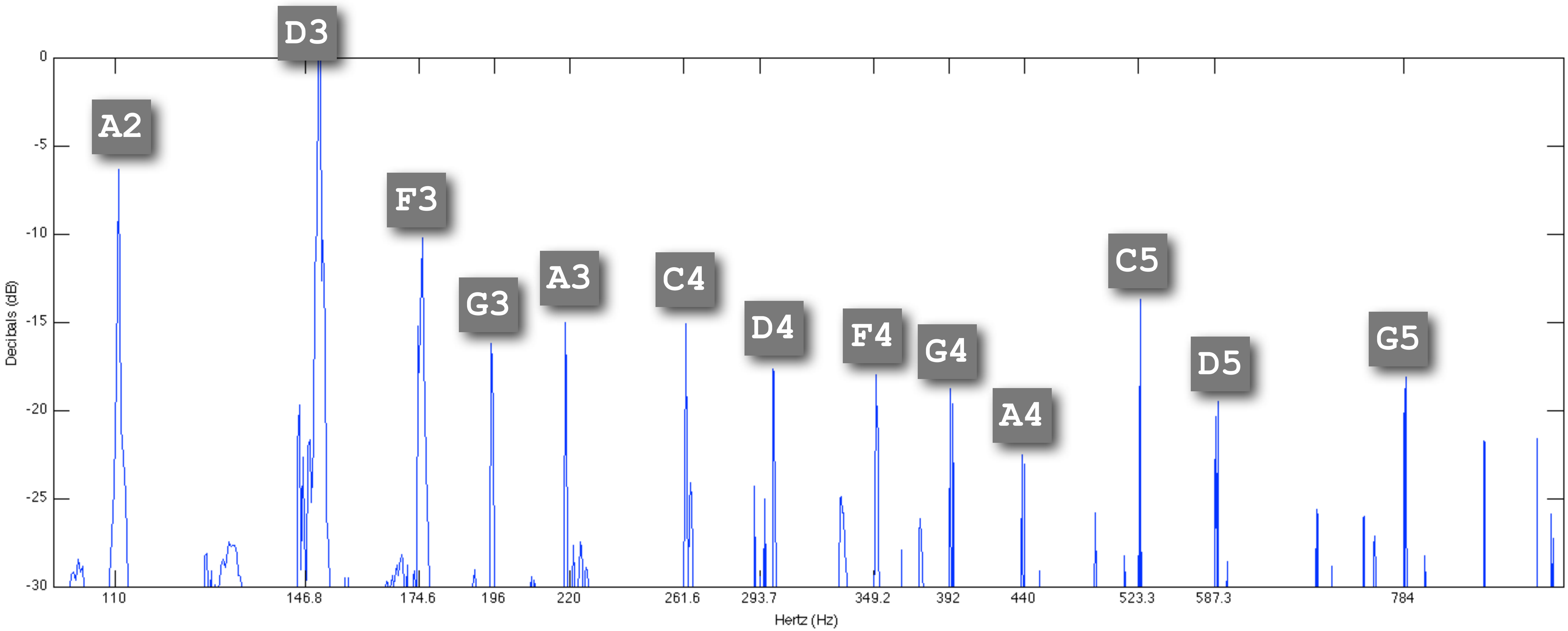
Some Detective Work



Some Detective Work

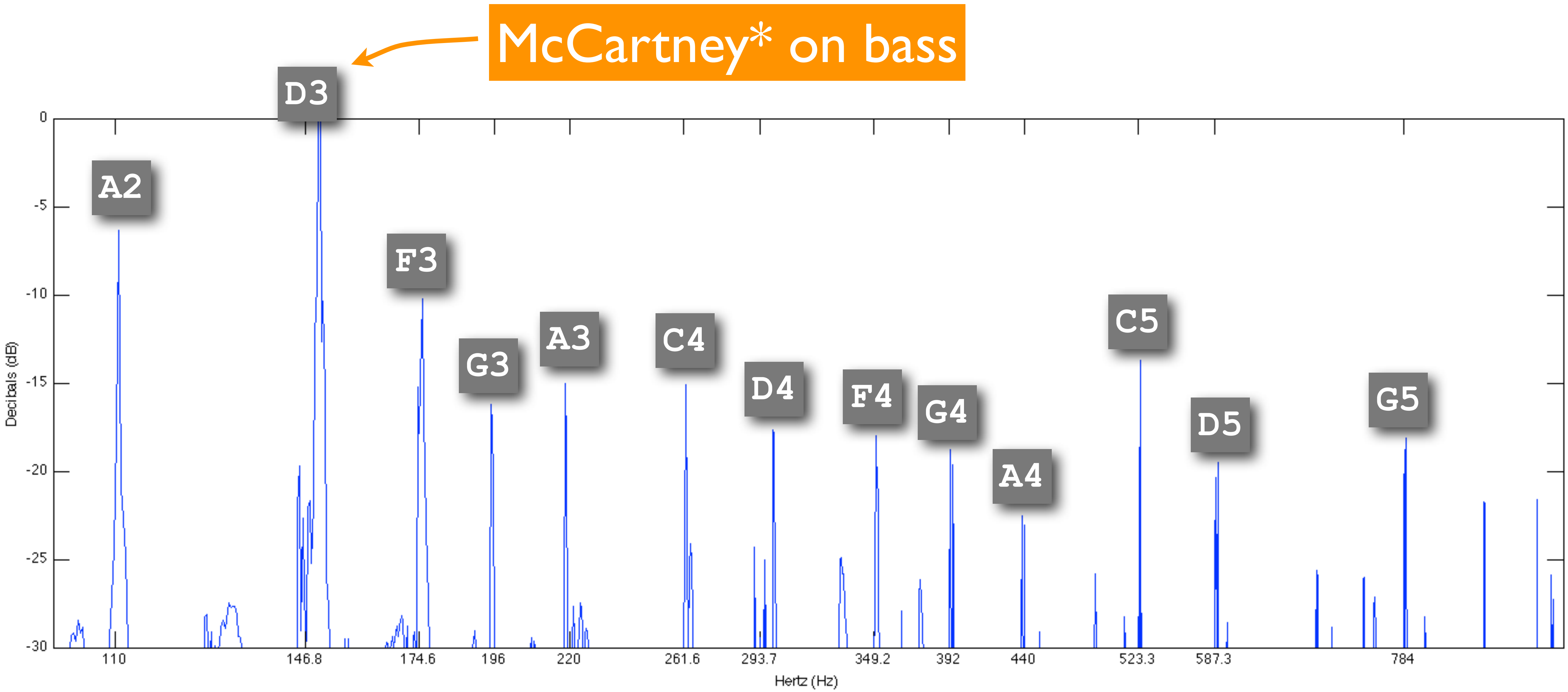


Some Detective Work



F major pentatonic scale / D bass

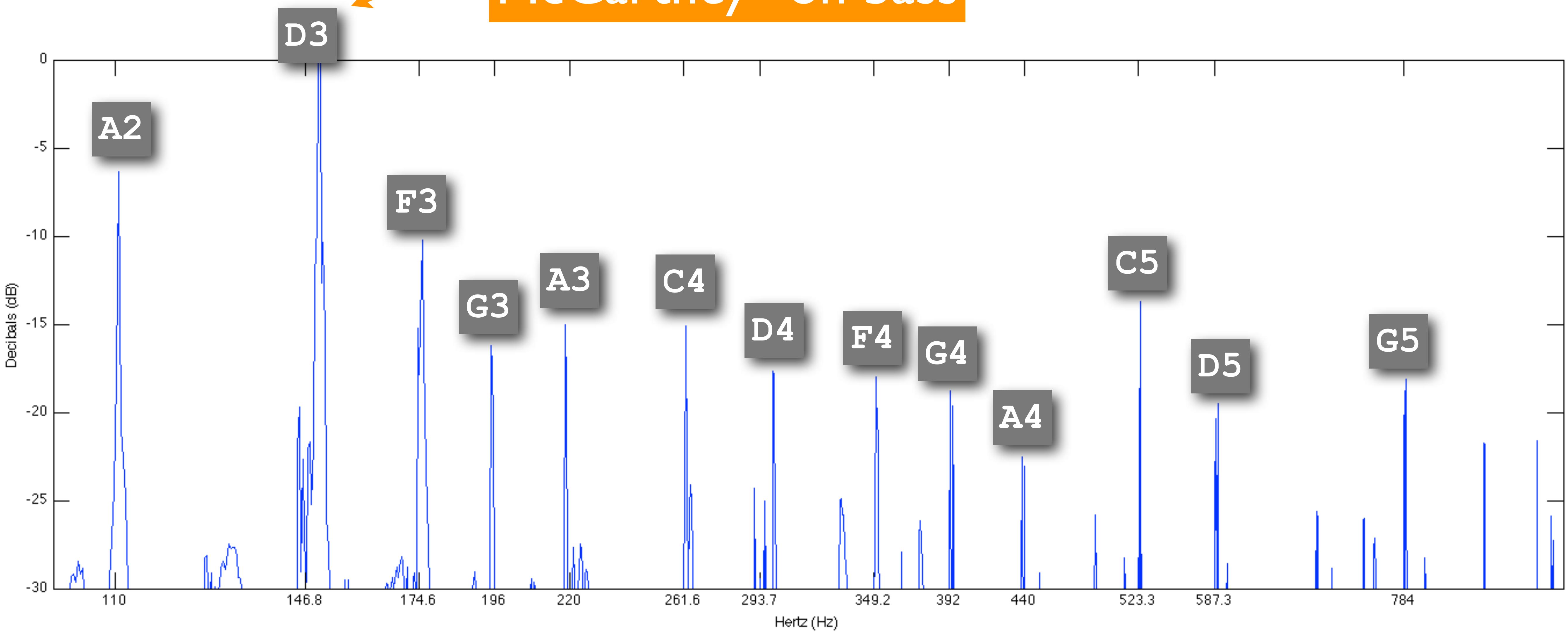
Some Detective Work



F major pentatonic scale / D bass

Some Detective Work

McCartney* on bass



F major pentatonic scale / D bass

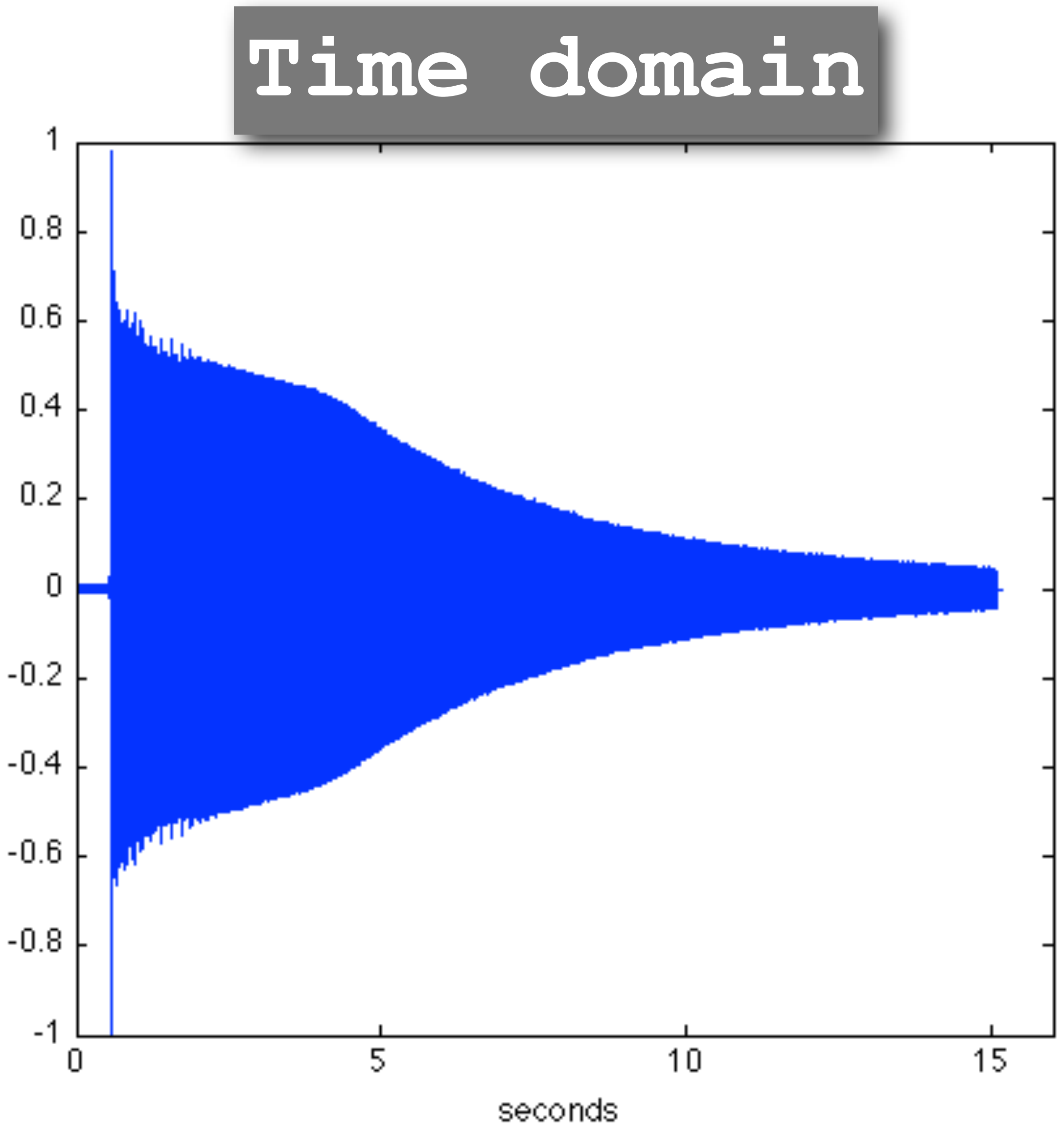
* = favorite Beatle

II. Of Tone and Timbre

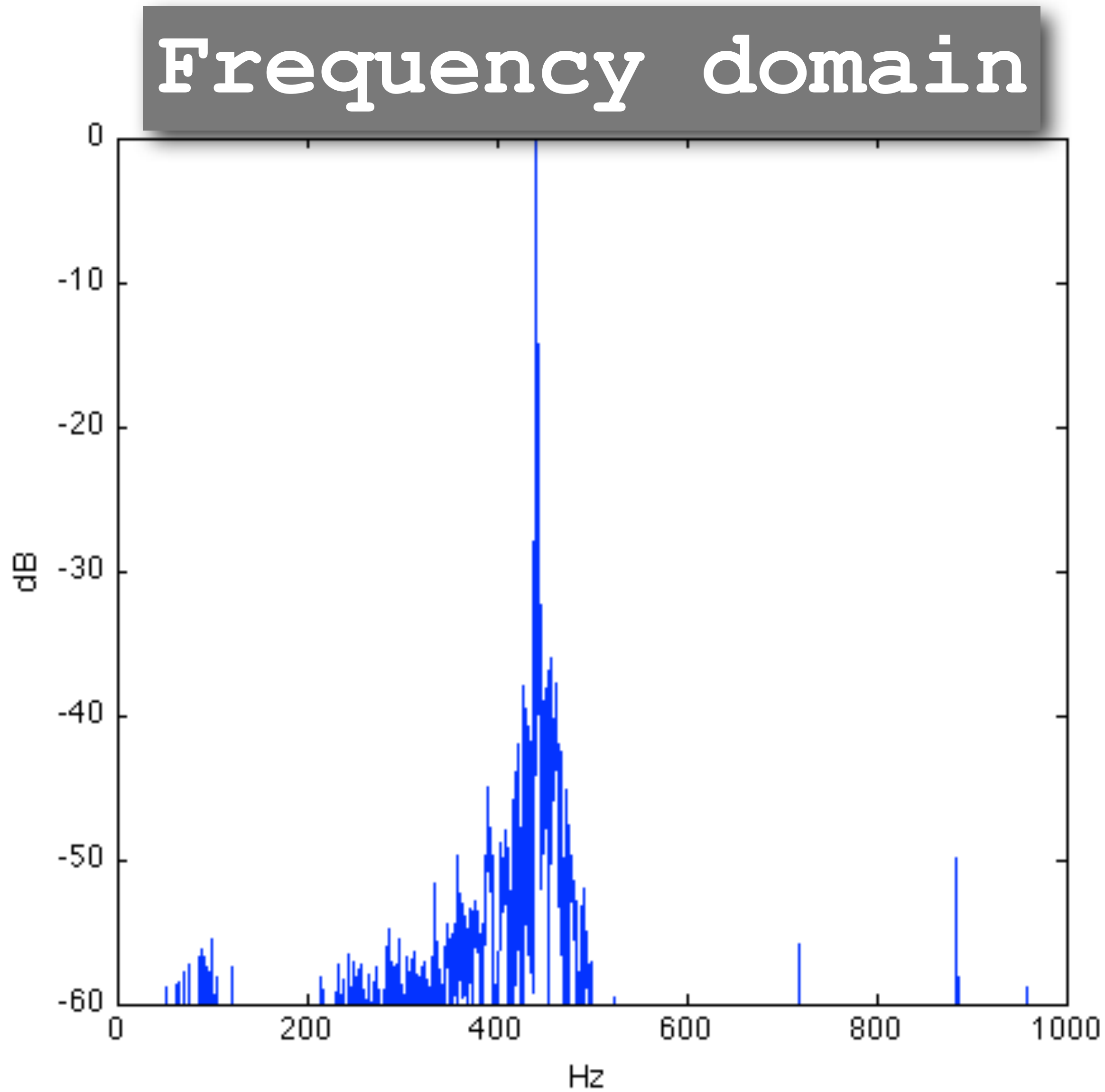
Pure Tones

Pure Tones

Pure Tones

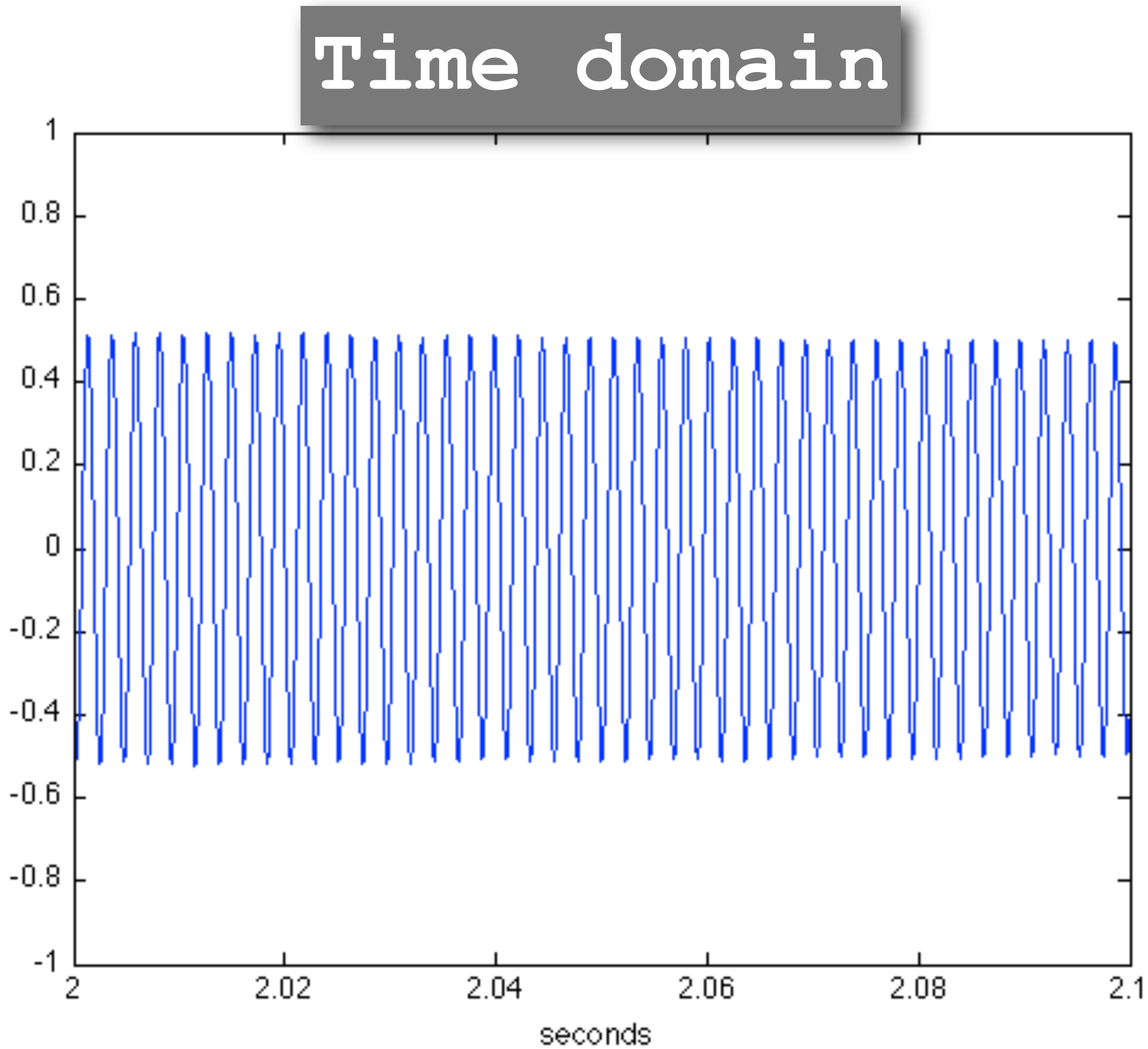


Time domain
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
seconds

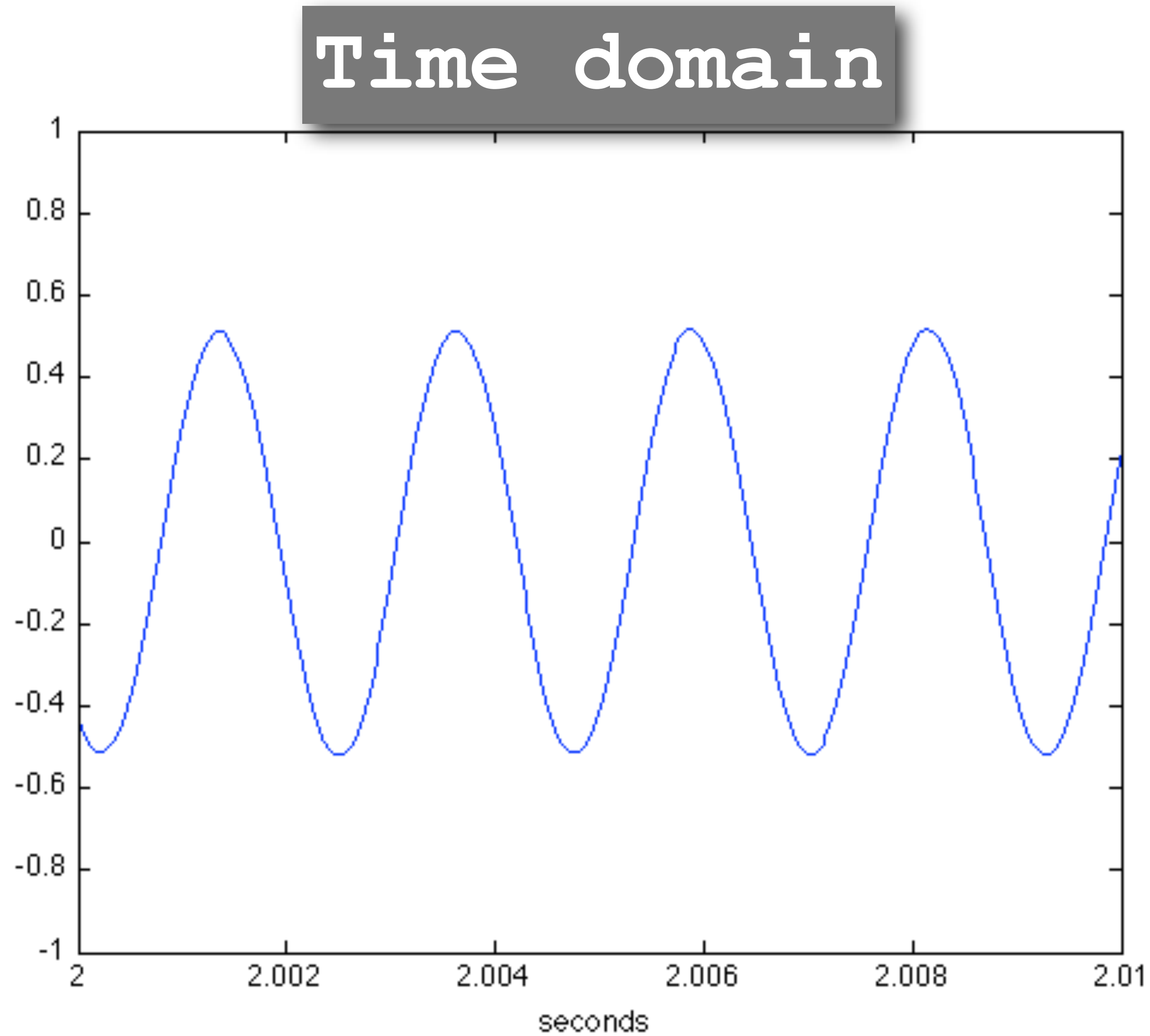


Frequency domain
0
-10
-20
-30
-40
-50
-60
dB
0
200
400
600
800
1000
Hz

Pure Tones


Time domain
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
2
2.02
2.04
2.06
2.08
2.1
seconds

Pure Tones



Time domain
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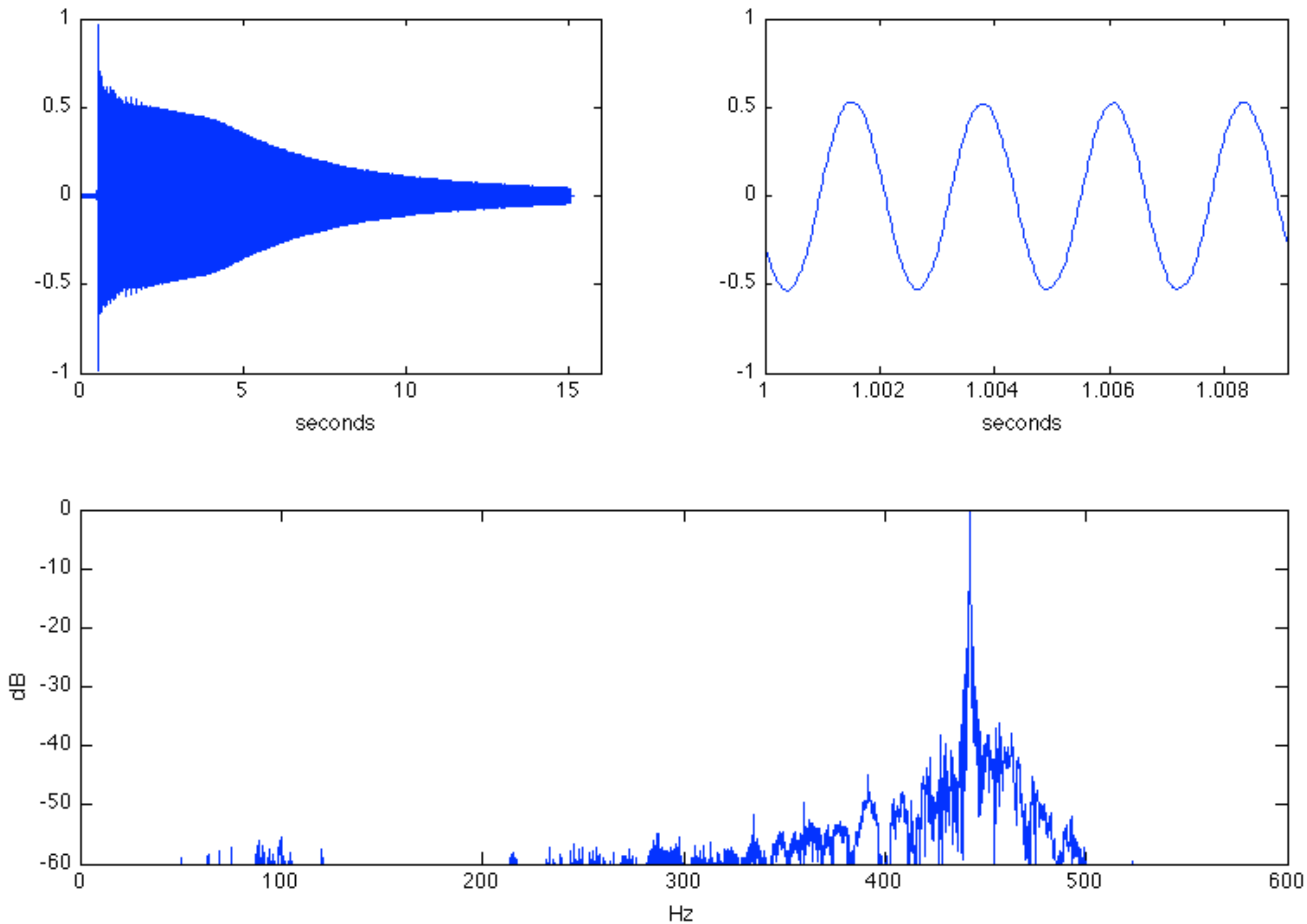
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
2
2.002
2.004
2.006
2.008
2.01
seconds

Pure Tones


1
0.5
0
-0.5
-1
0
5
10
15
seconds
1
0.5
0
-0.5
-1
1
1.002
1.004
1.006
1.008
seconds
0
-10
-20
-30
-40
-50
-60
dB
0
100
200
300
400
500
600
Hz

Pure Tones

- Definition: a <u>pure tone</u> is a function of the form

$$s(t) = A\cos\left(2\pi f_0 t + \phi\right)$$

Pure Tones

- Definition: a <u>pure tone</u> is a function of the form

amplitude

$$s(t) = A\cos\left(2\pi f_0 t + \phi\right)$$

Pure Tones

- Definition: a <u>pure tone</u> is a function of the form

amplitude
frequency

$$s(t) = \underbrace{A}_{\text{amplitude}} \cos\left(2\pi \underbrace{f_0}_{\text{frequency}} t + \phi\right)$$

Pure Tones

- Definition: a <u>pure tone</u> is a function of the form

amplitude frequency phase

$$s(t) = \underbrace{A}_{\text{amplitude}} \cos\left(2\pi \underbrace{f_0}_{\text{frequency}} t + \underbrace{\phi}_{\text{phase}}\right)$$

Pure Tones

- Definition: a <u>pure tone</u> is a function of the form

amplitude

frequency

phase

$$s(t) = A\cos\left(2\pi f_0 t + \phi\right)$$

$$= a\cos\left(2\pi f_0 t\right) + b\sin\left(2\pi f_0 t\right)$$

Simulated Tuning Fork

- Simulate a tuning fork by enveloping a pure tone with a time-varying amplitude:

$$s(t) = A(t)\cos\left(2\pi f_0 t + \phi\right)$$

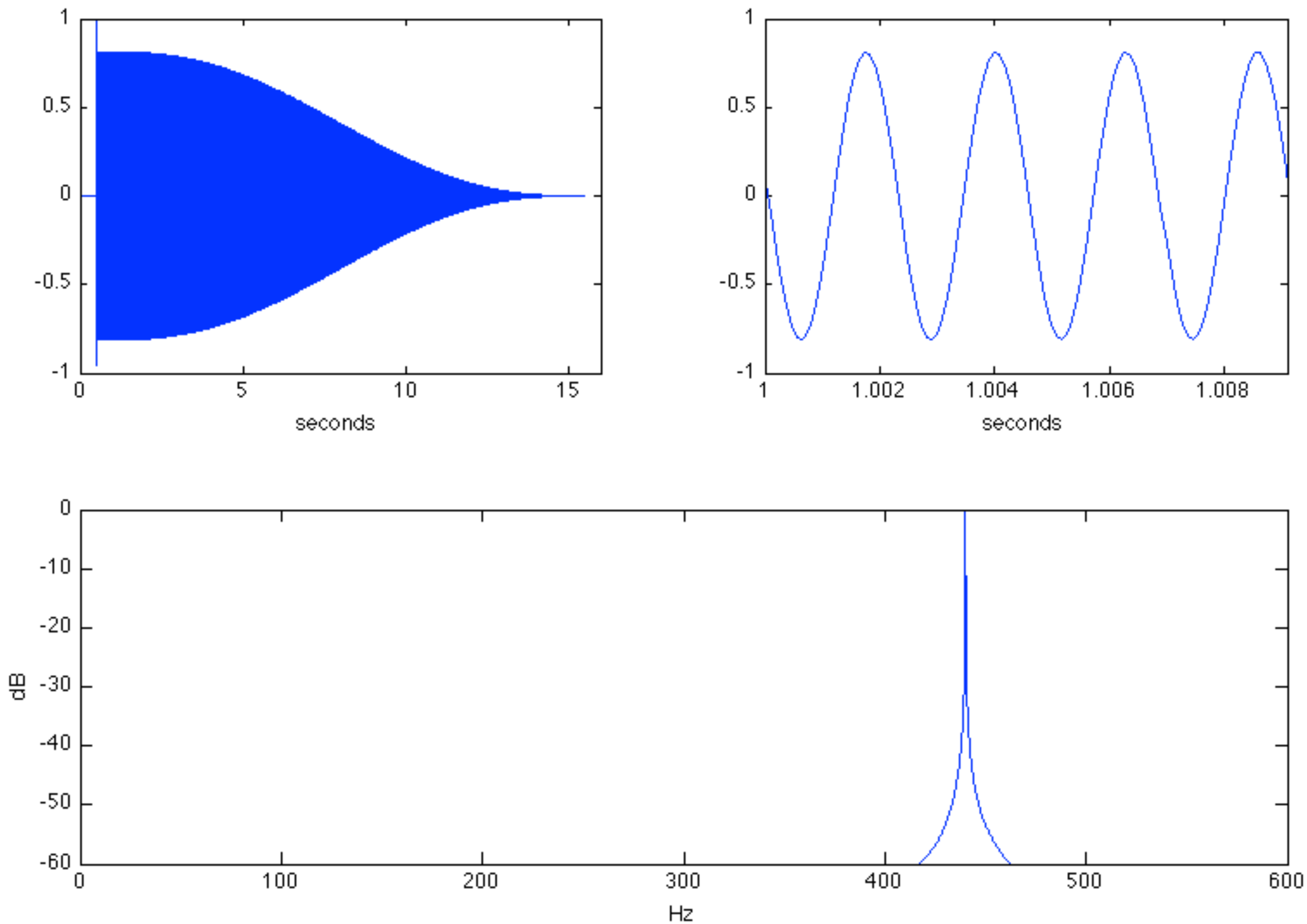
Simulated Tuning Fork

- Simulate a tuning fork by enveloping a pure tone with a time-varying amplitude:

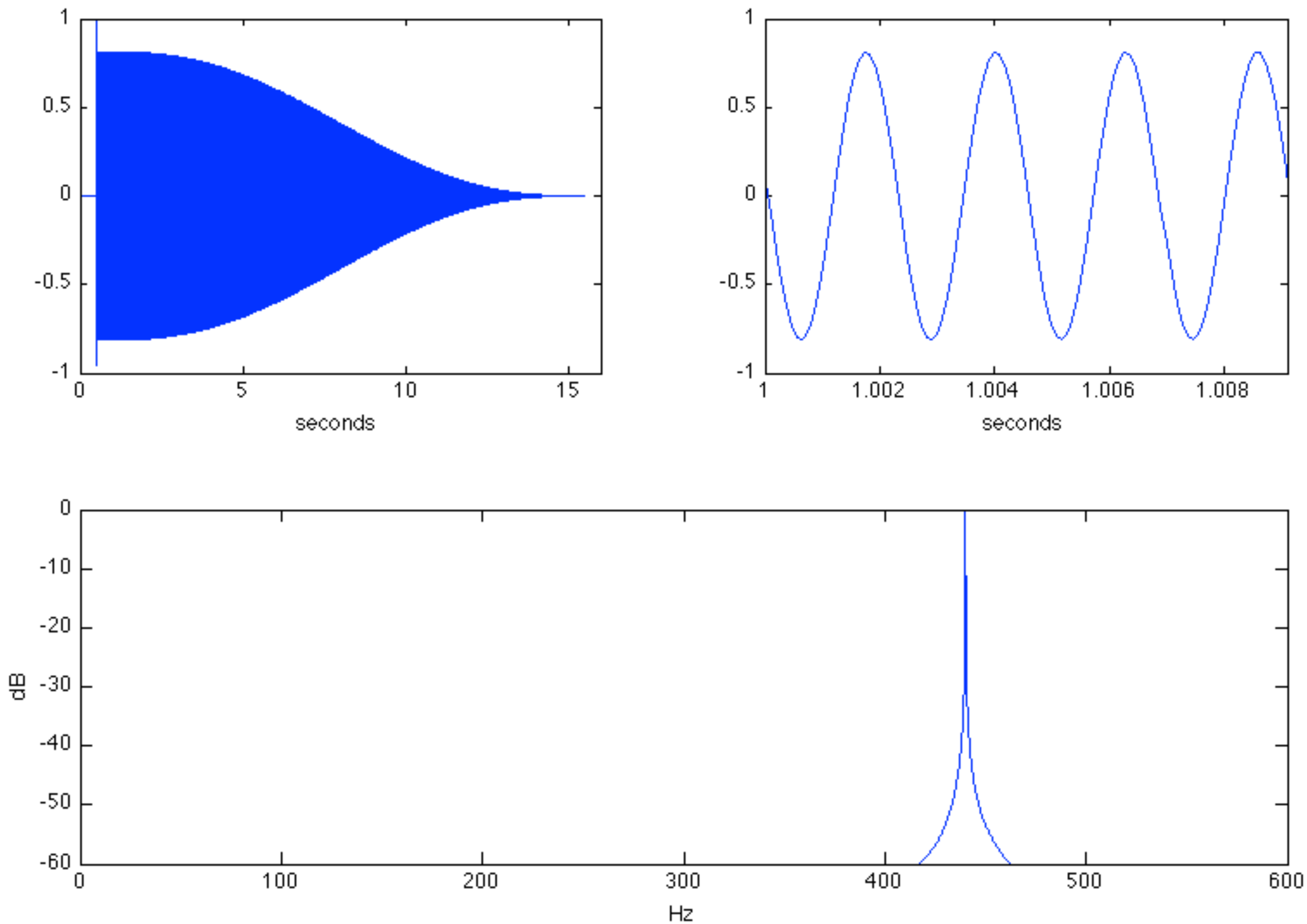
$$\underbrace{A(t)}_{\texttt{time-varying amplitude}}$$

$$s(t) = A(t)\cos\left(2\pi f_0 t + \phi\right)$$

Simulated Tuning Fork

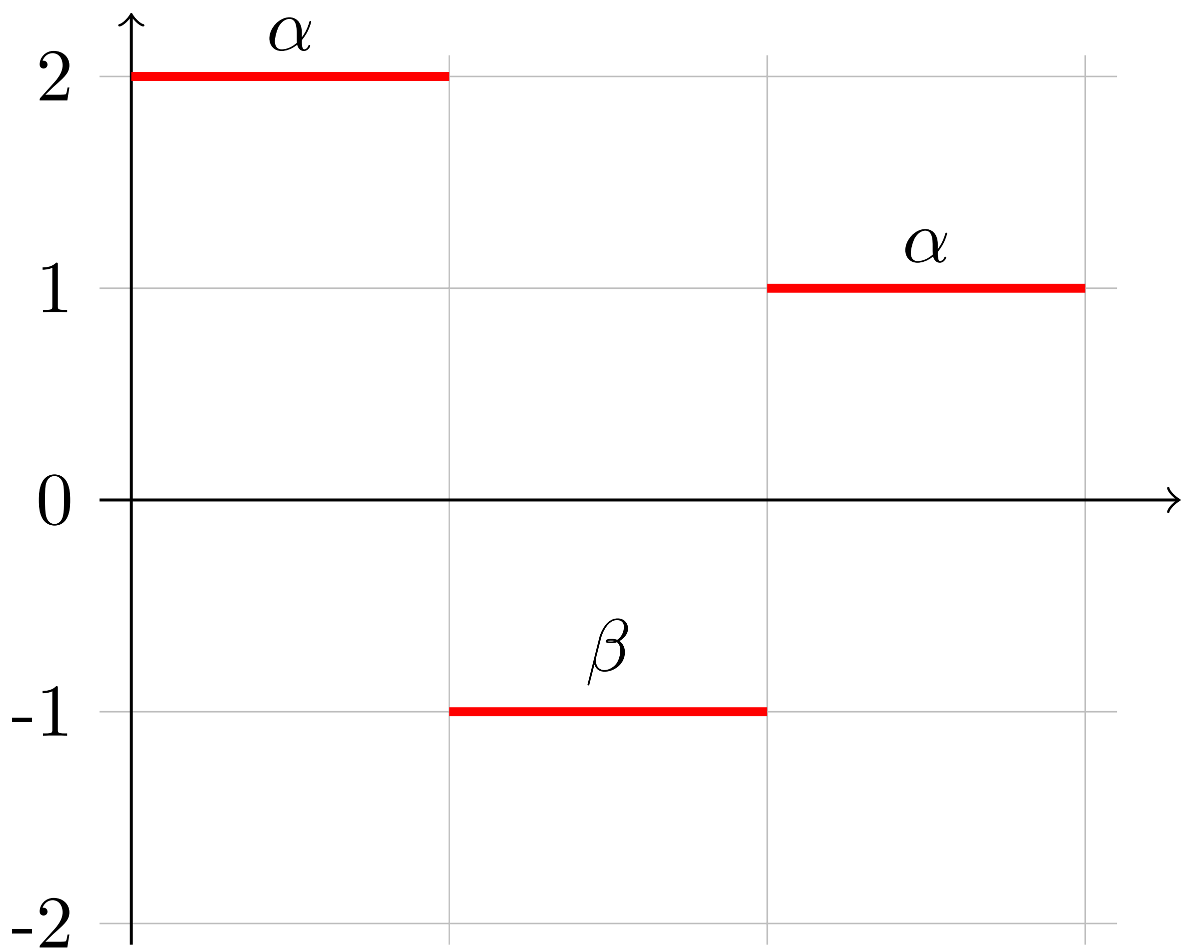


Simulated Tuning Fork

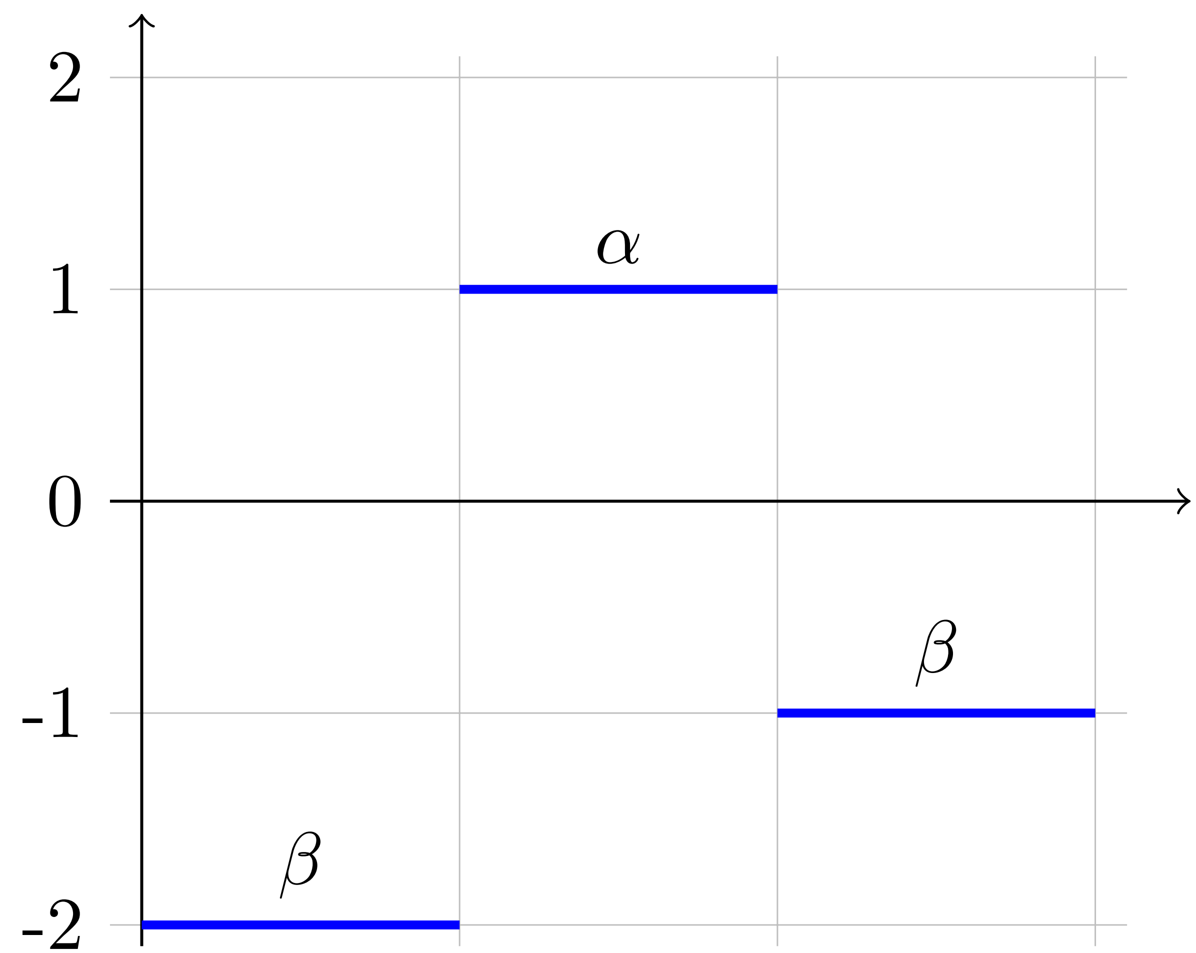


Opus 1: Duet for Pure Tones

Pattern α

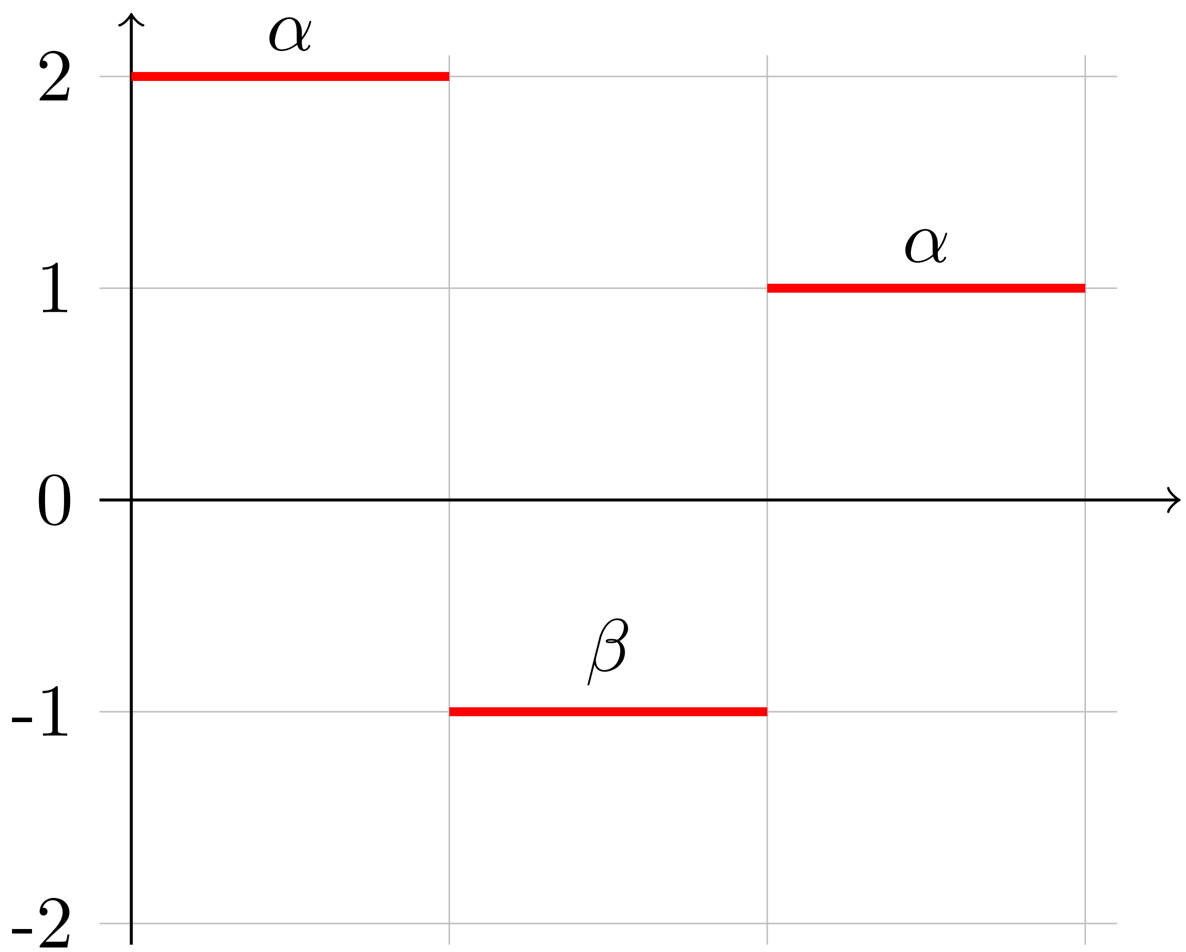


Pattern β

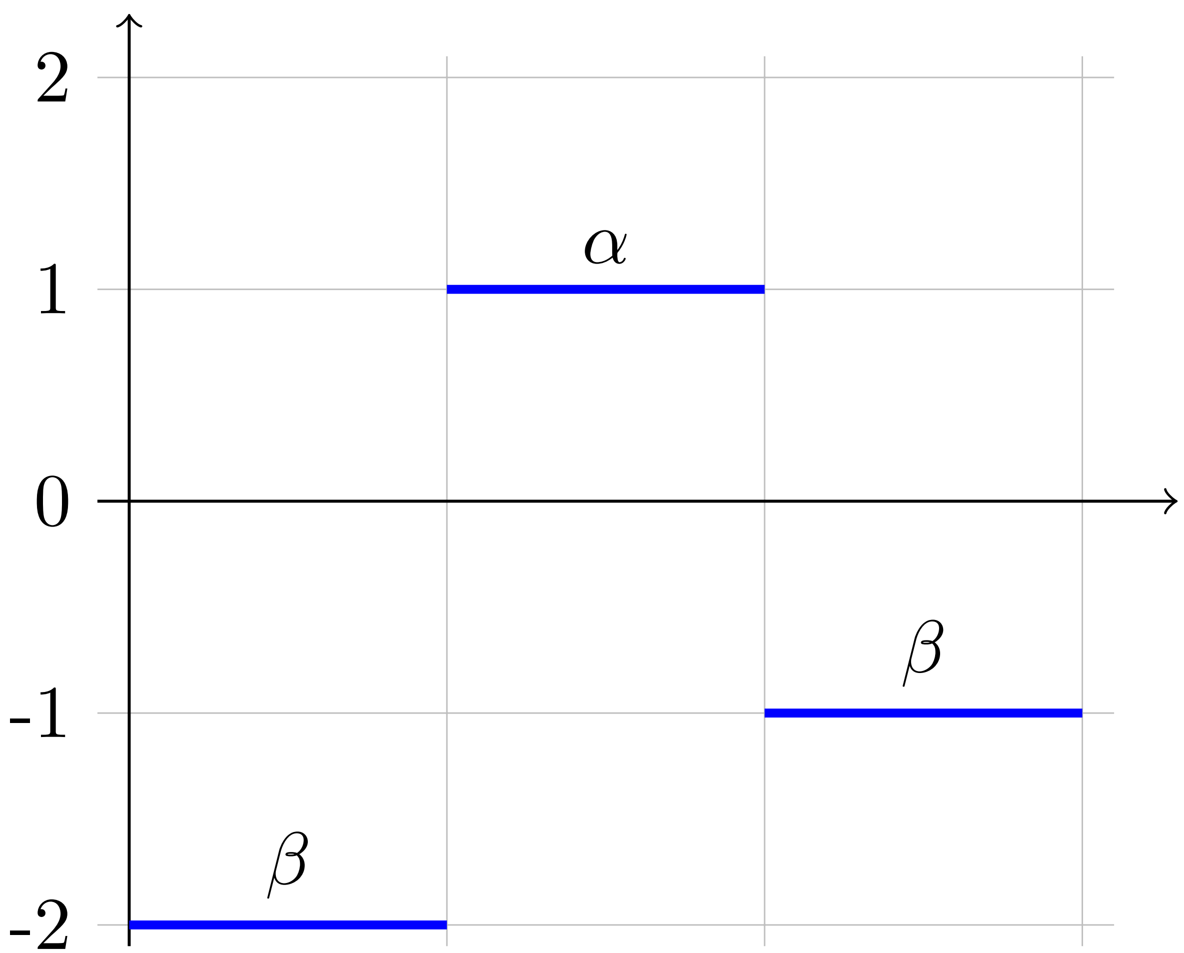


Opus 1: Duet for Pure Tones

Pattern α



Pattern β



Not so pure tones...

Not so pure tones...

Timbre

Timbre

- Timbre is the intrinsic "sound quality" of a musical note or sound.

Timbre

- Timbre is the intrinsic “sound quality” of a musical note or sound.
- Name that timbre!

Timbre

- Timbre is the intrinsic "sound quality" of a musical note or sound.
- Name that timbre!

Timbre

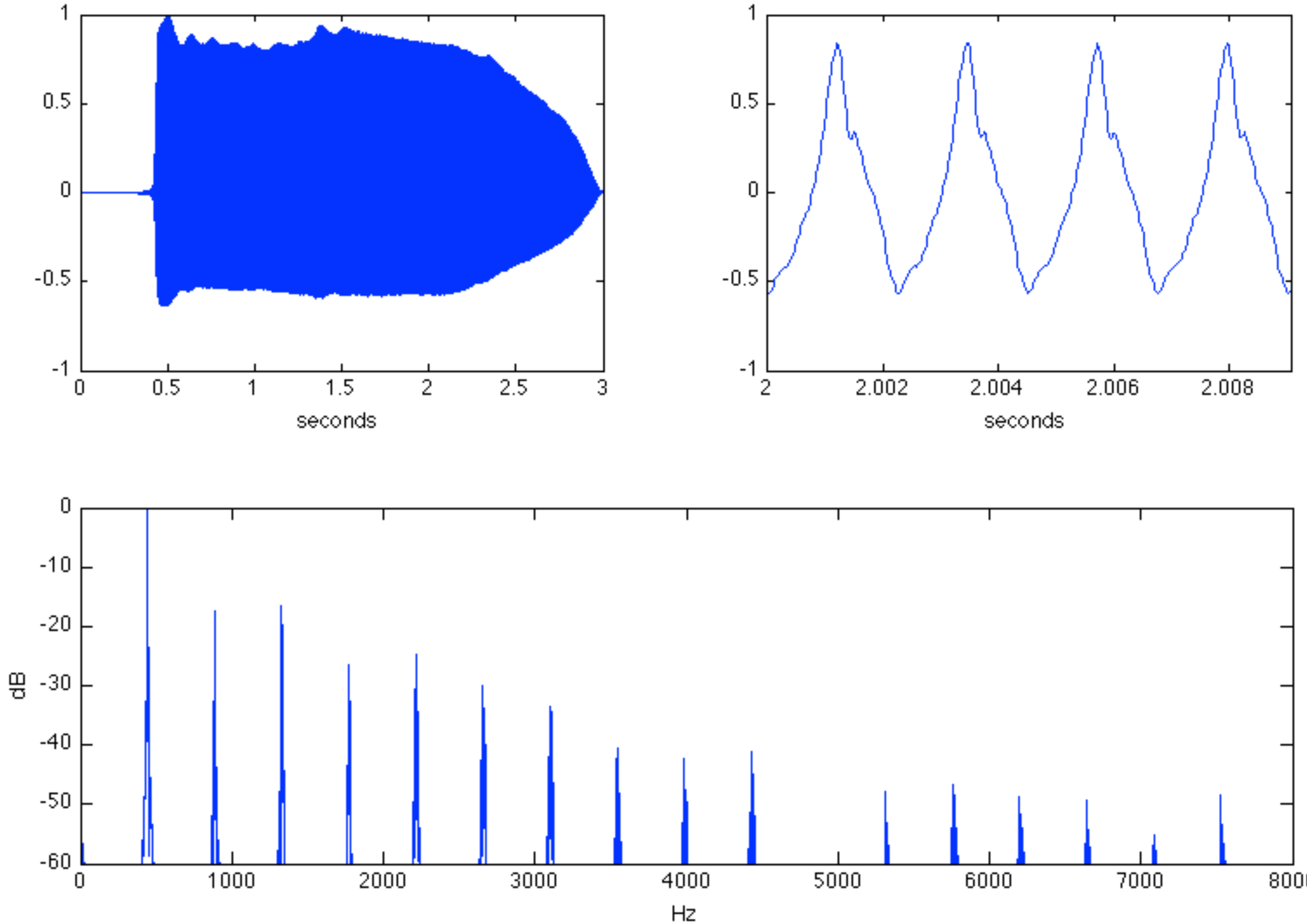
- Timbre is the intrinsic “sound quality” of a musical note or sound.

- Name that timbre!

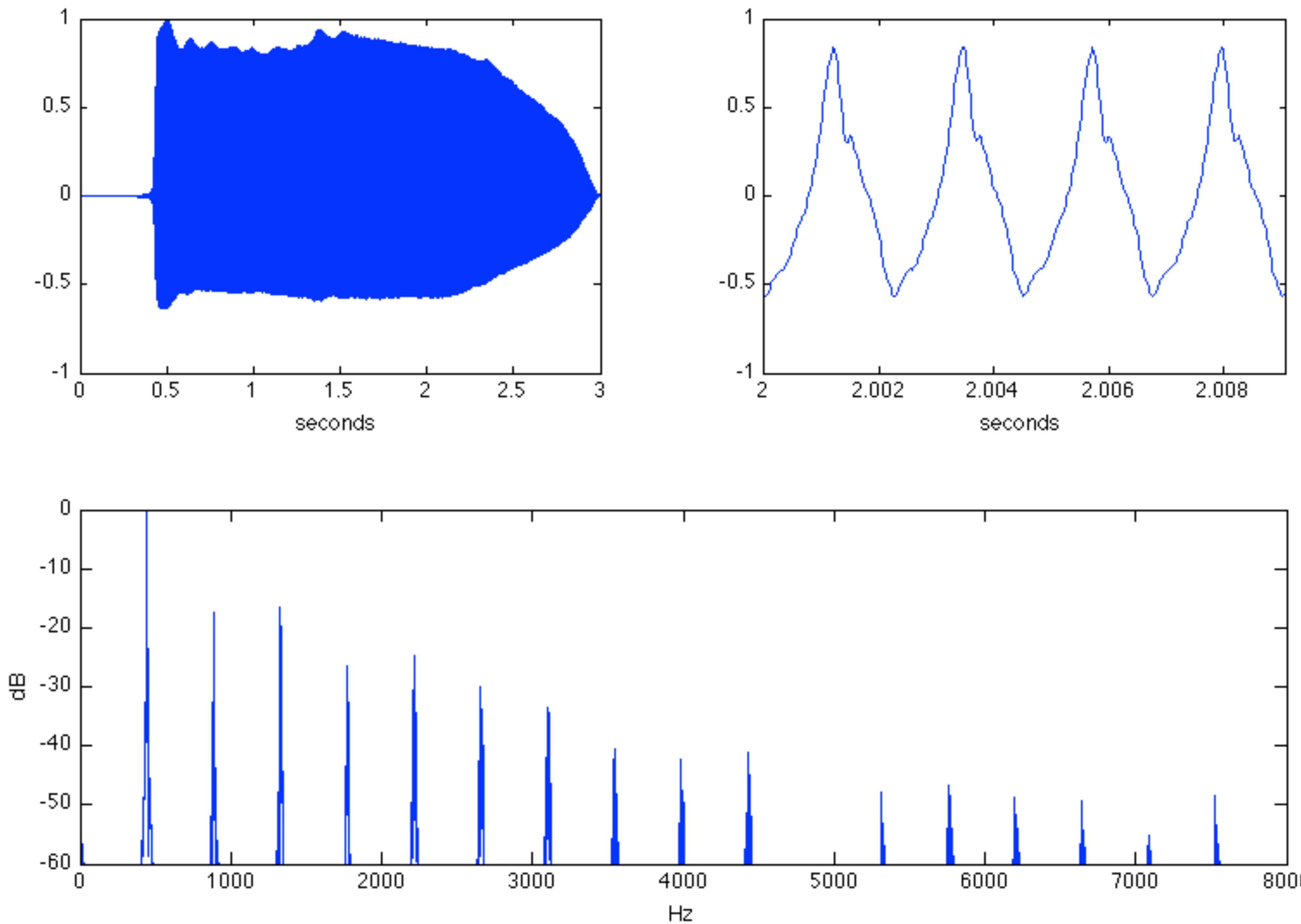
Timbre

- Timbre is the intrinsic "sound quality" of a musical note or sound.
- Name that timbre!
- What determines an instrument's characteristic timbre?

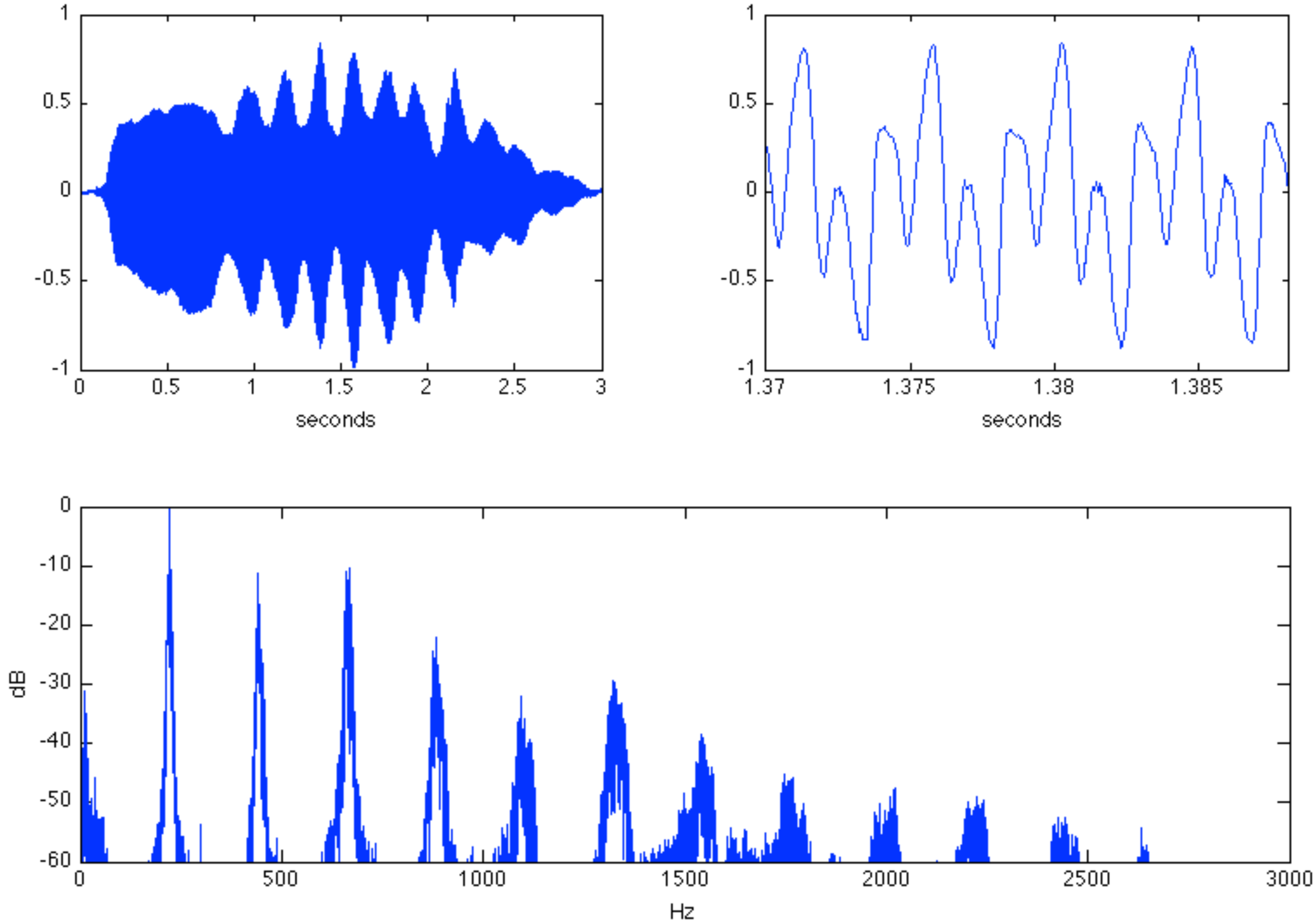
Alto Saxophone (A4)



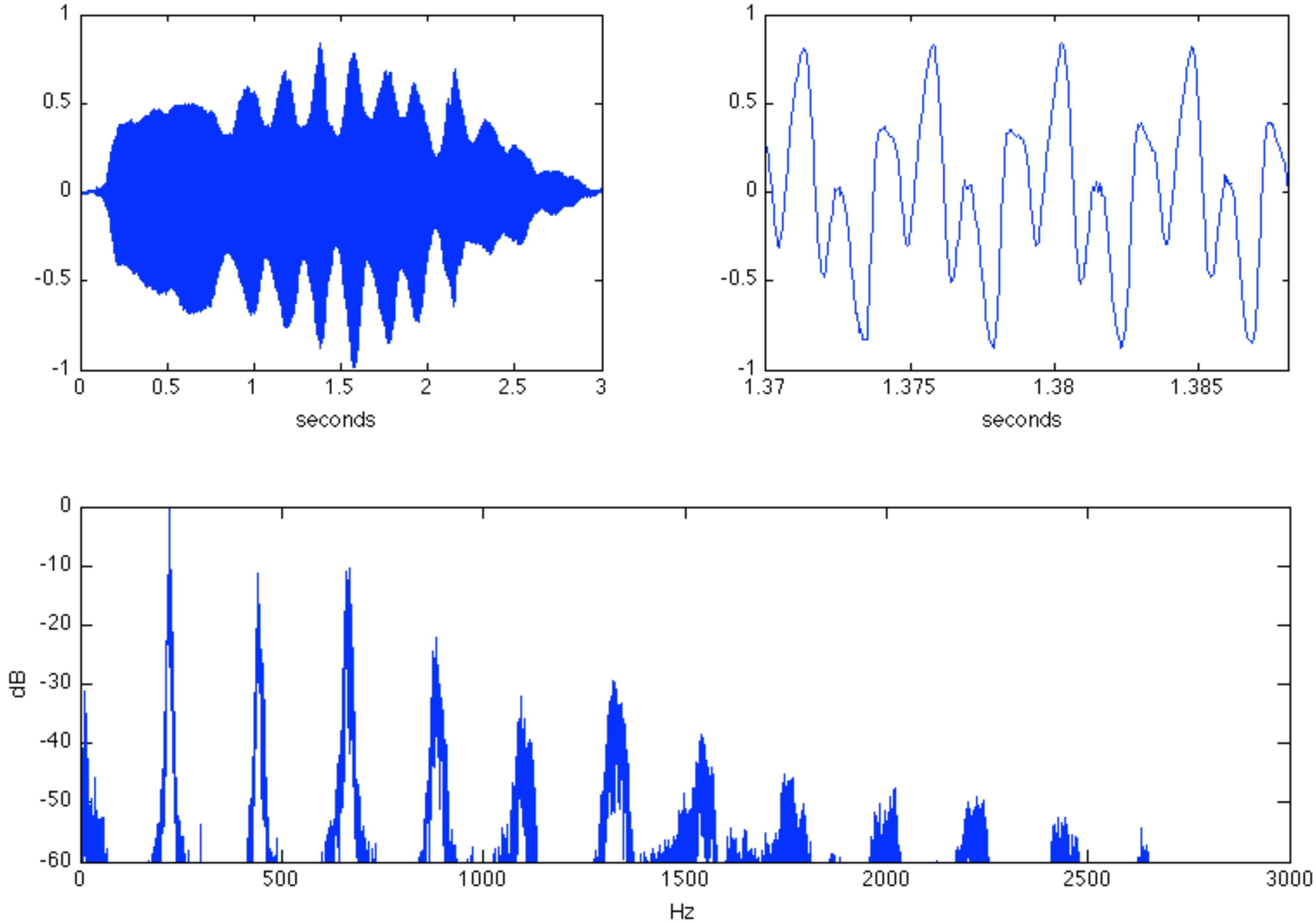
Alto Saxophone (A4)



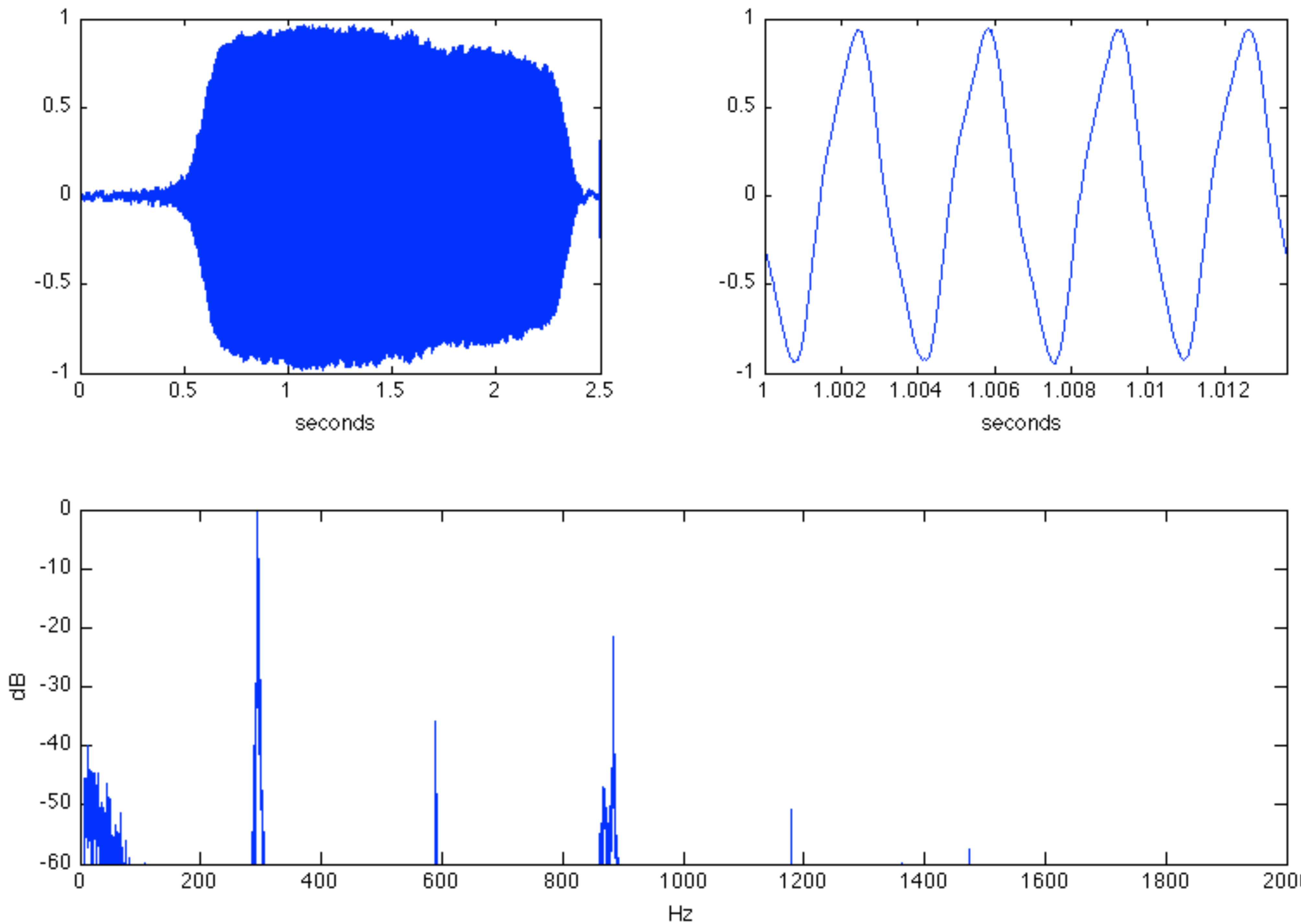
Bass Flute (A3)



Bass Flute (A3)

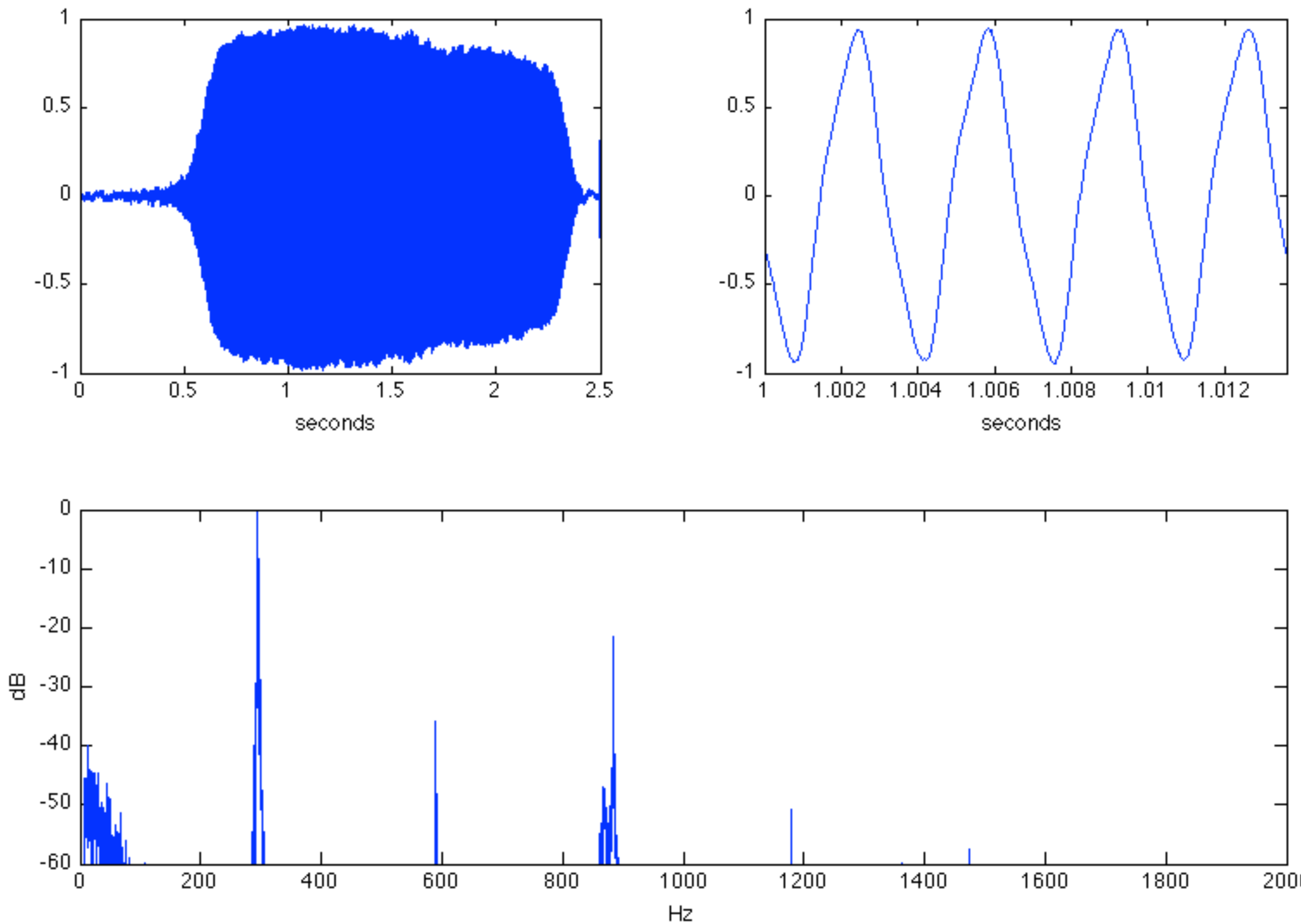


Clarinet (D4)



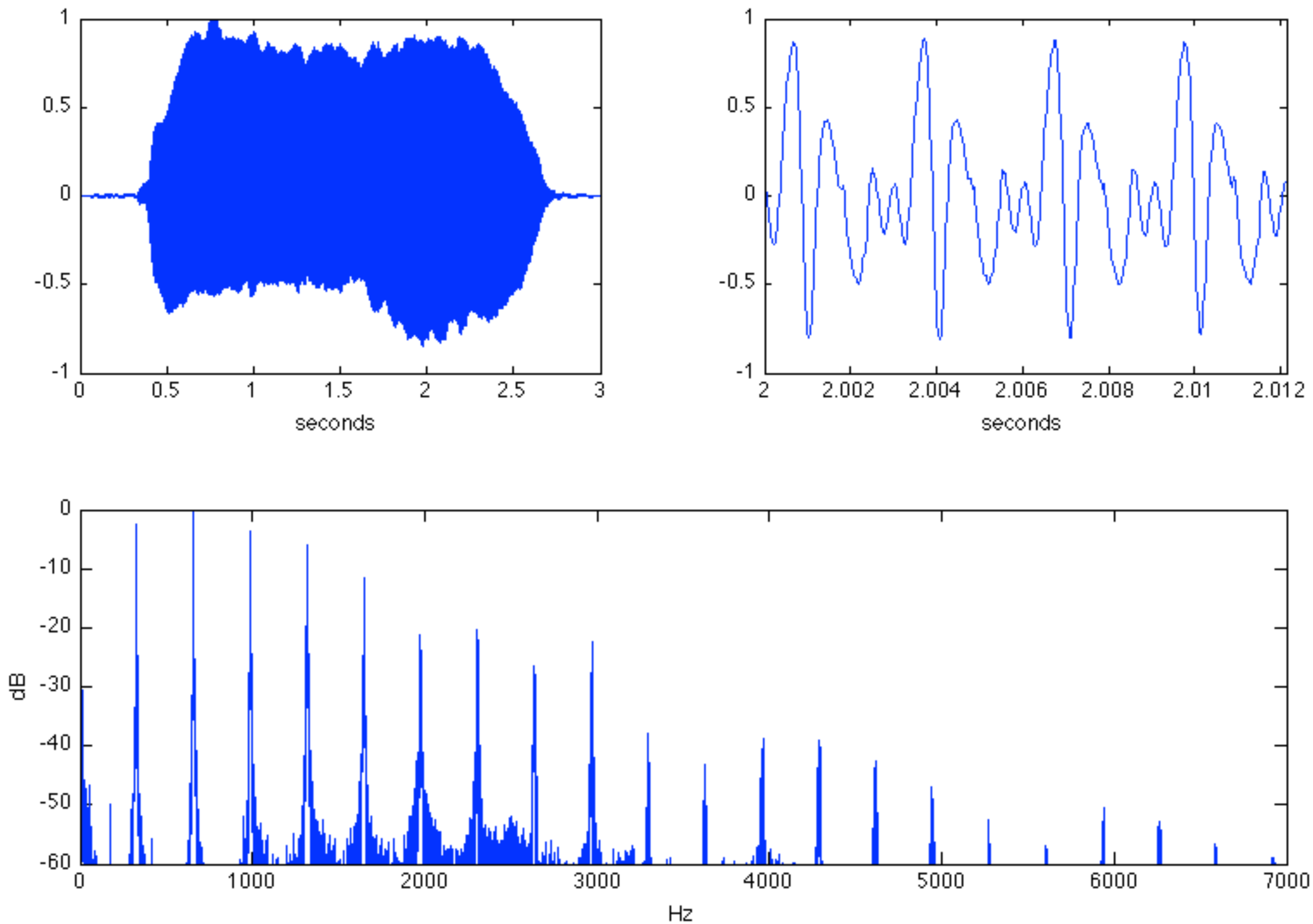
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
seconds
1
0.5
0
-0.5
-1
1
1.002
1.004
1.006
1.008
1.01
1.012
seconds
0
-10
-20
-30
-40
-50
-60
dB
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Hz

Clarinet (D4)

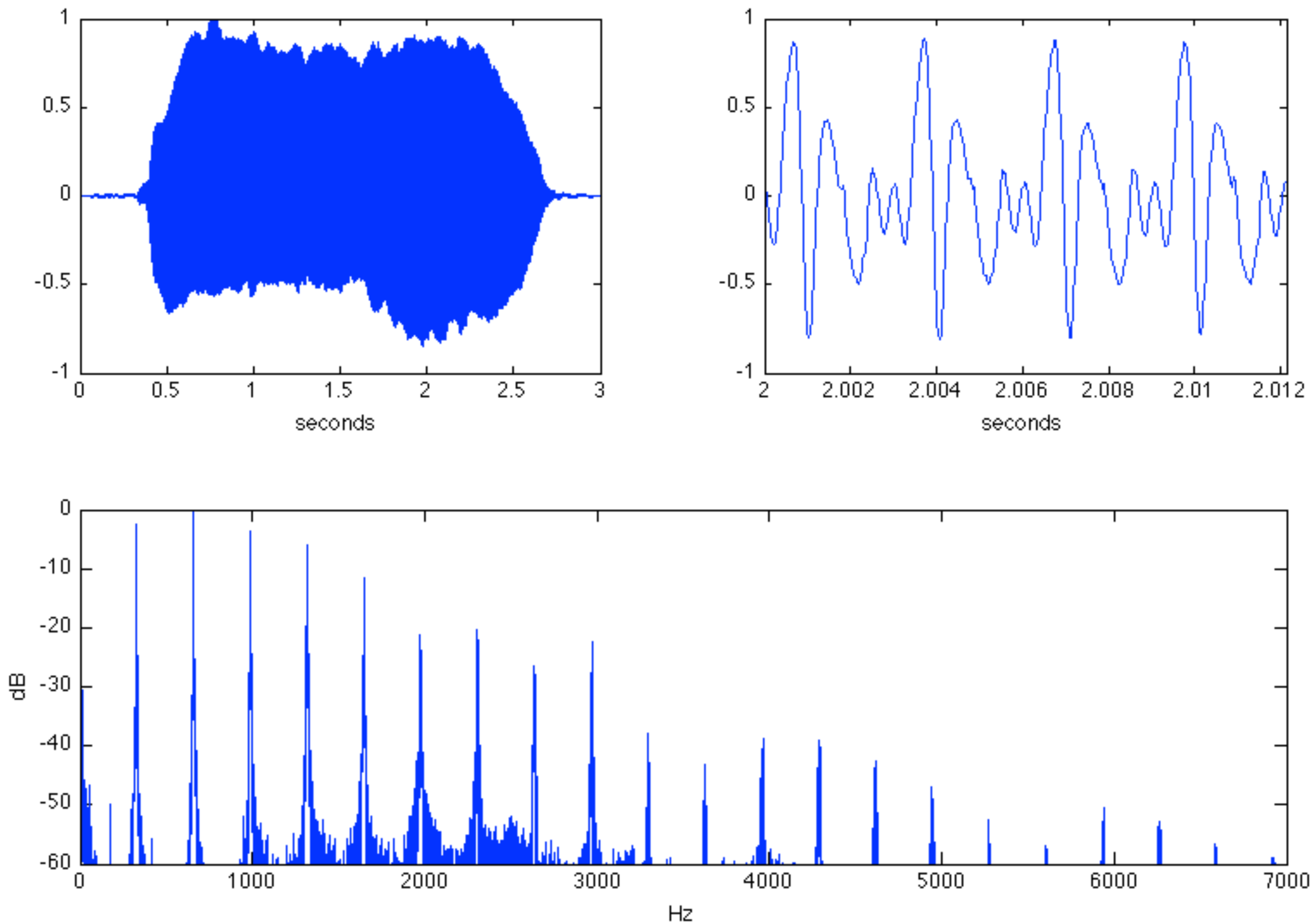


1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
seconds
1
0.5
0
-0.5
-1
1
1.002
1.004
1.006
1.008
1.01
1.012
seconds
0
-10
-20
-30
-40
-50
-60
dB
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Hz

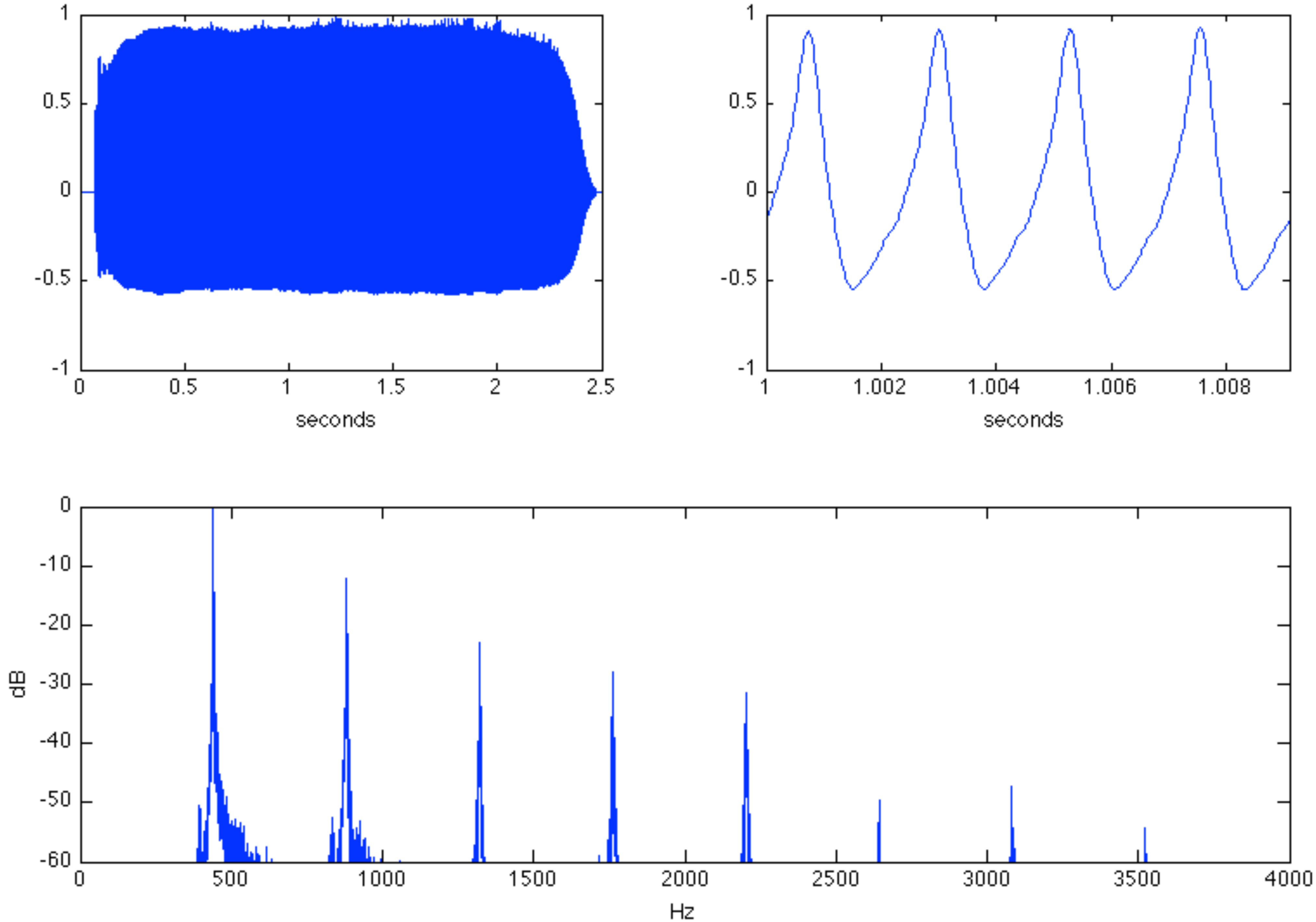
Flute (E4)


1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
seconds
1
0.5
0
-0.5
-1
2
2.002
2.004
2.006
2.008
2.01
2.012
seconds
0
-10
-20
-30
-40
-50
-60
dB
0
1000
2000
3000
4000
5000
6000
7000
Hz

Flute (E4)

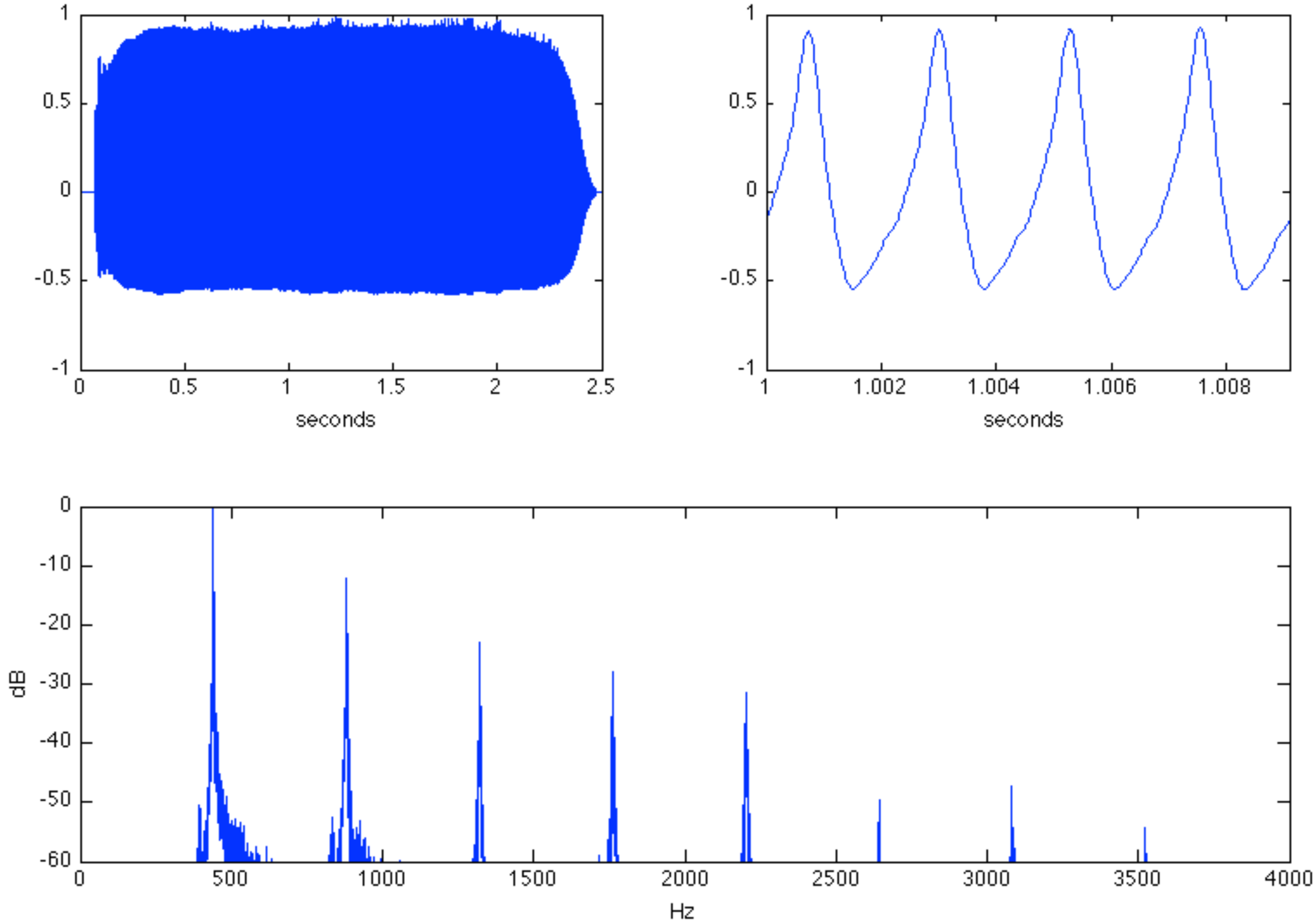

1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
seconds
1
0.5
0
-0.5
-1
2
2.002
2.004
2.006
2.008
2.01
2.012
seconds
0
-10
-20
-30
-40
-50
-60
dB
0
1000
2000
3000
4000
5000
6000
7000
Hz

Horn (A4)



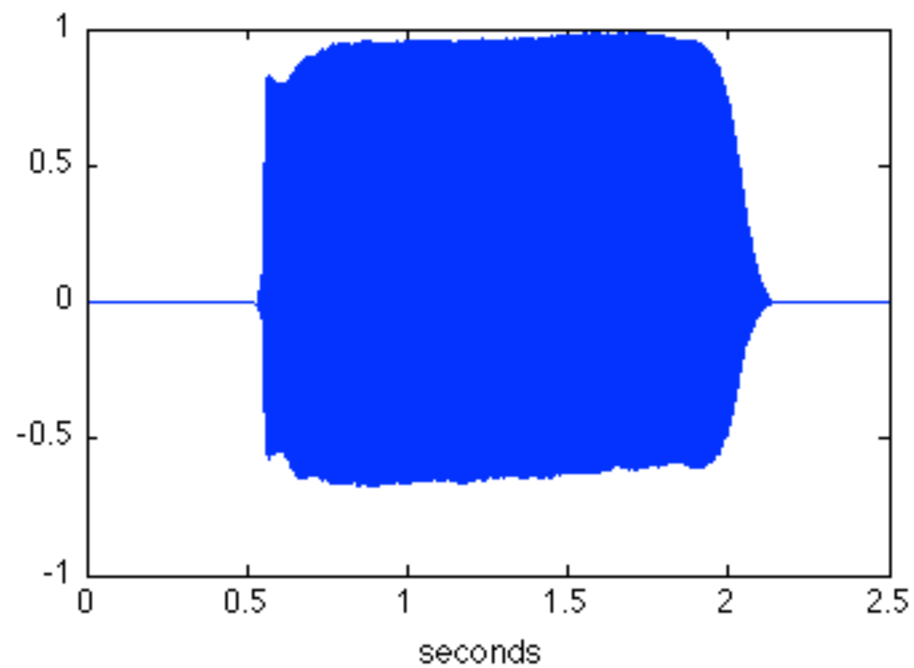
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
seconds
1
0.5
0
-0.5
-1
1
1.002
1.004
1.006
1.008
seconds
0
-10
-20
-30
-40
-50
-60
dB
0
500
1000
1500
2000
2500
3000
3500
4000
Hz

Horn (A4)

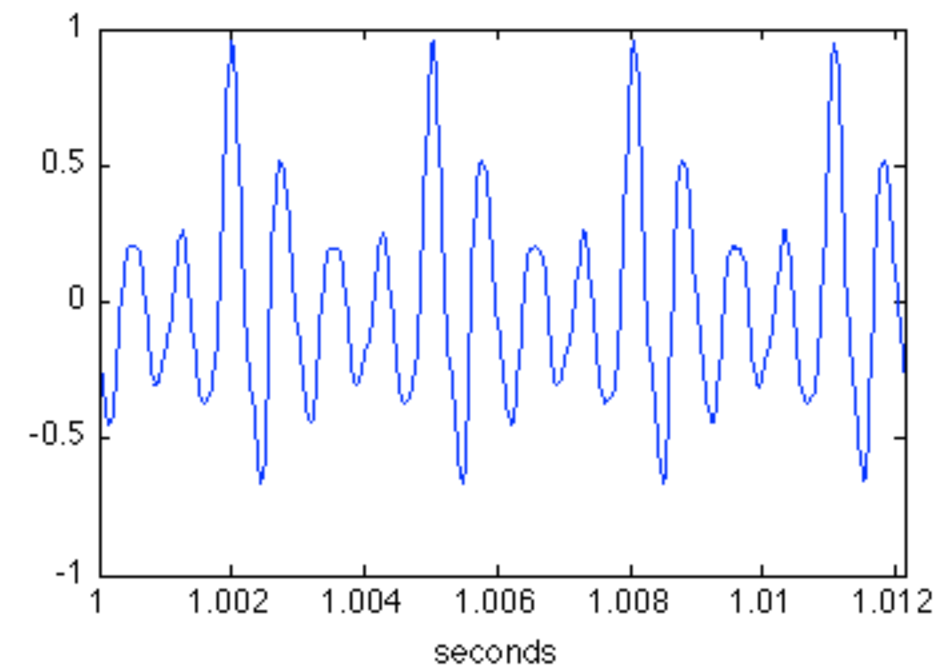


1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
seconds
1
0.5
0
-0.5
-1
1
1.002
1.004
1.006
1.008
seconds
0
-10
-20
-30
-40
-50
-60
dB
0
500
1000
1500
2000
2500
3000
3500
4000
Hz

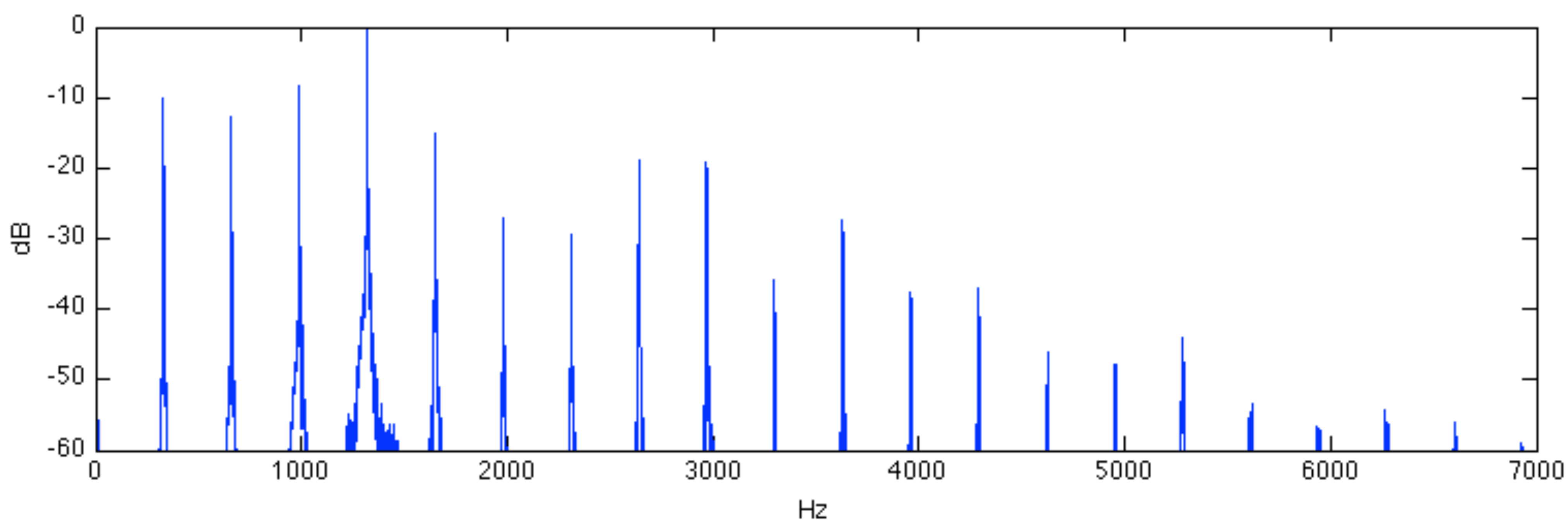
Oboe (E4)



1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
seconds

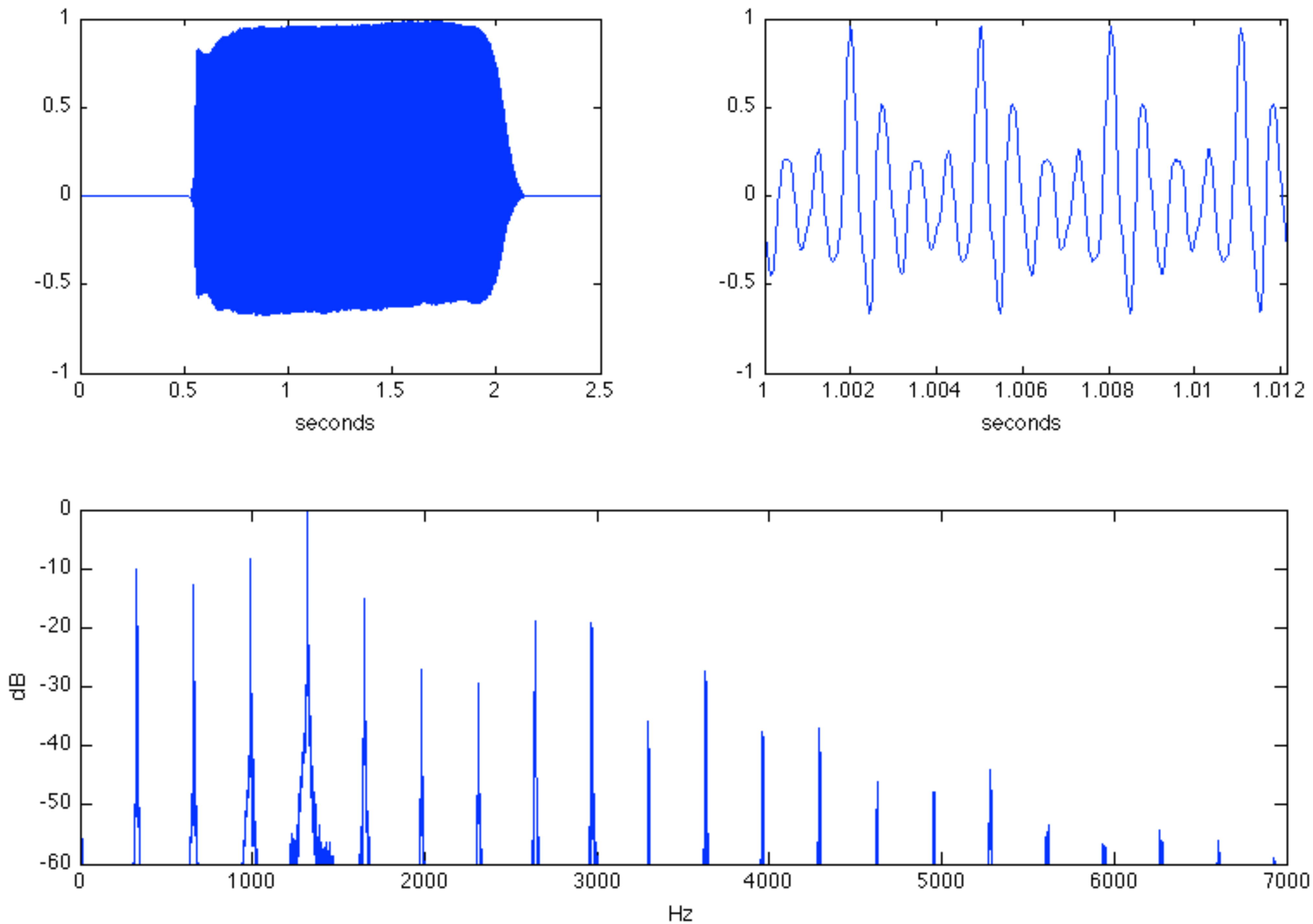


1
0.5
0
-0.5
-1
1
1.002
1.004
1.006
1.008
1.01
1.012
seconds



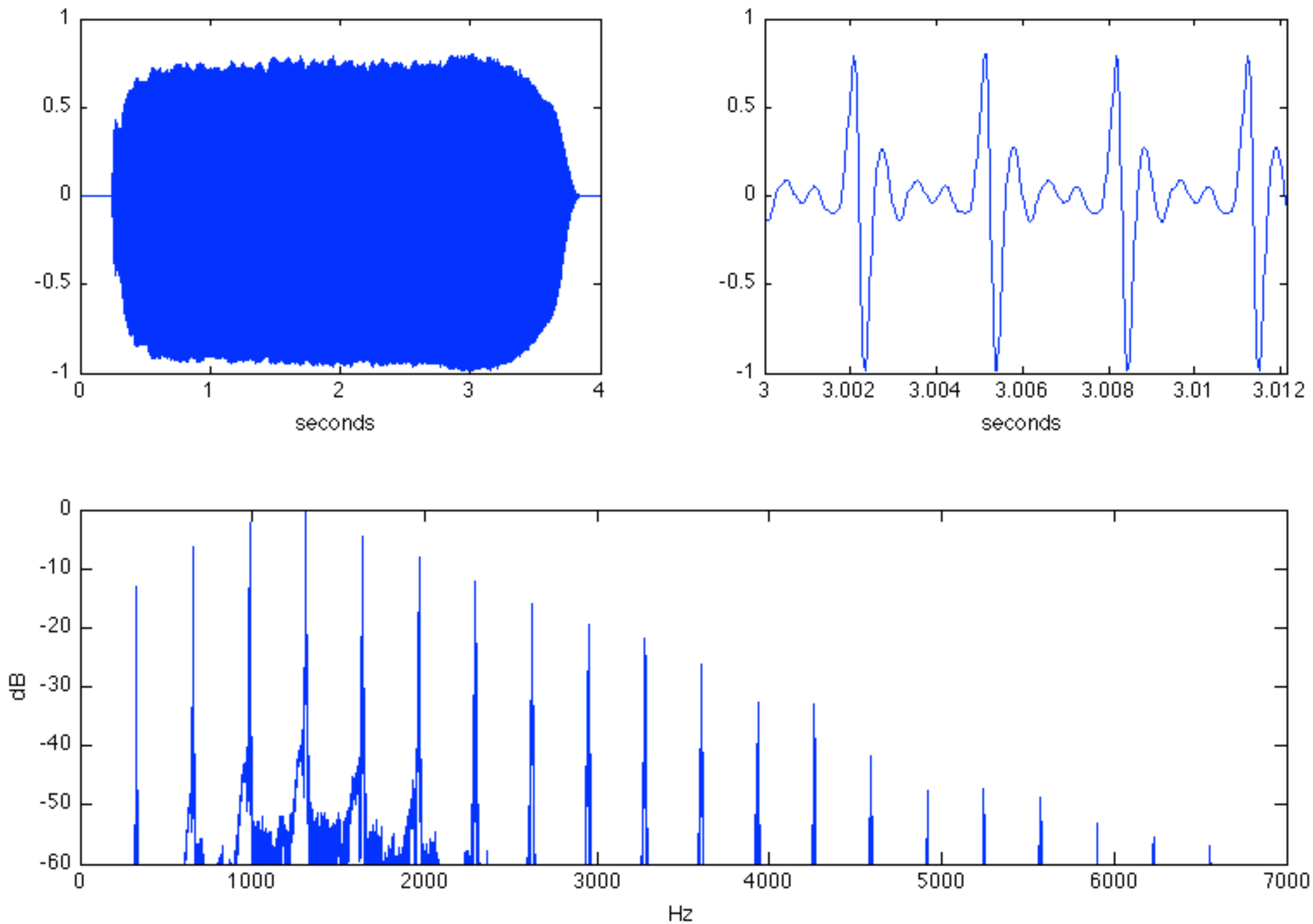
0
-10
-20
-30
-40
-50
-60
dB
0
1000
2000
3000
4000
5000
6000
7000
Hz

Oboe (E4)



1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
seconds
1
0.5
0
-0.5
-1
1
1.002
1.004
1.006
1.008
1.01
1.012
seconds
0
-10
-20
-30
-40
-50
-60
dB
0
1000
2000
3000
4000
5000
6000
7000
Hz

Trumpet (E4)


1
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seconds
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3
3.002
3.004
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3.008
3.01
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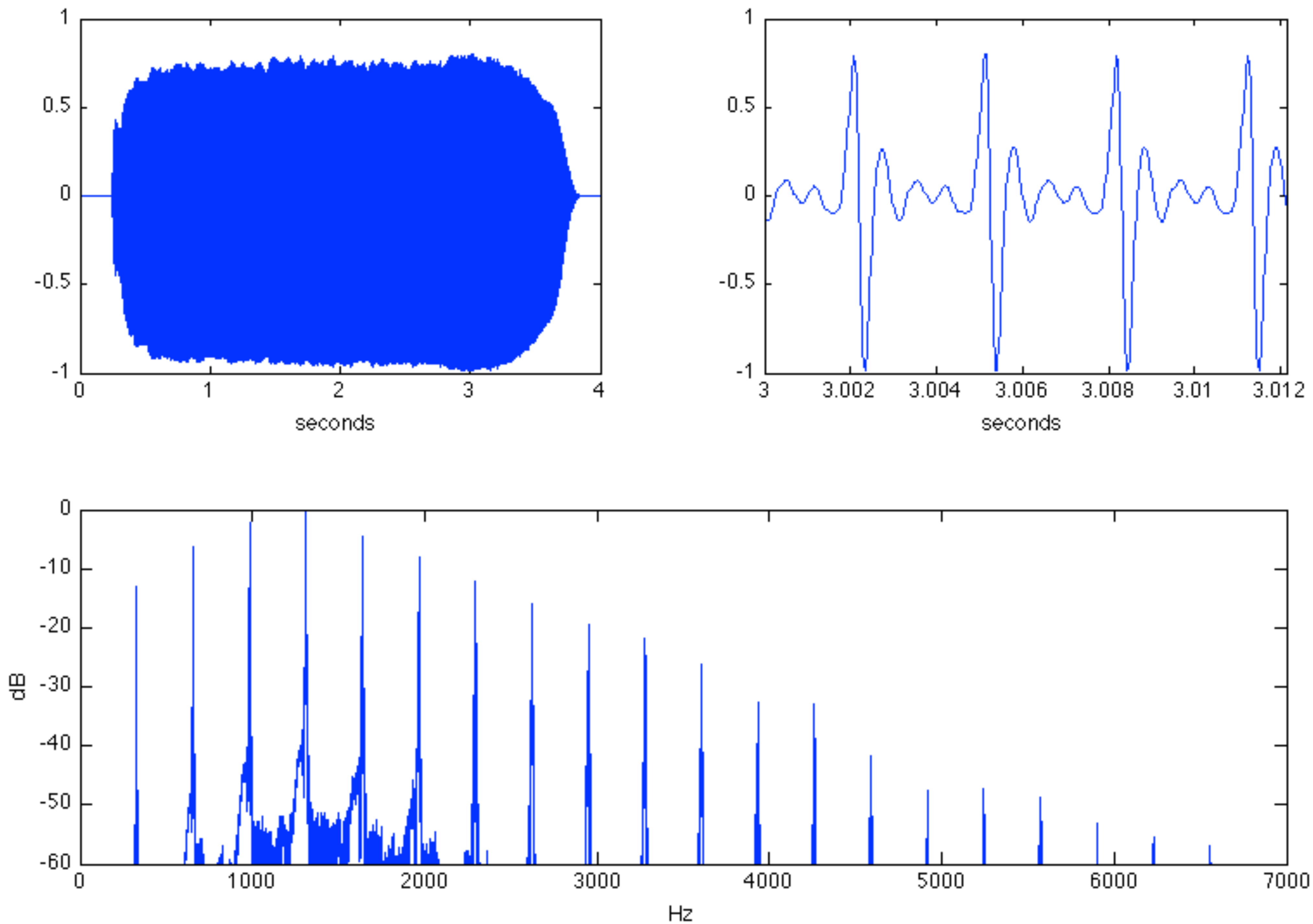

1
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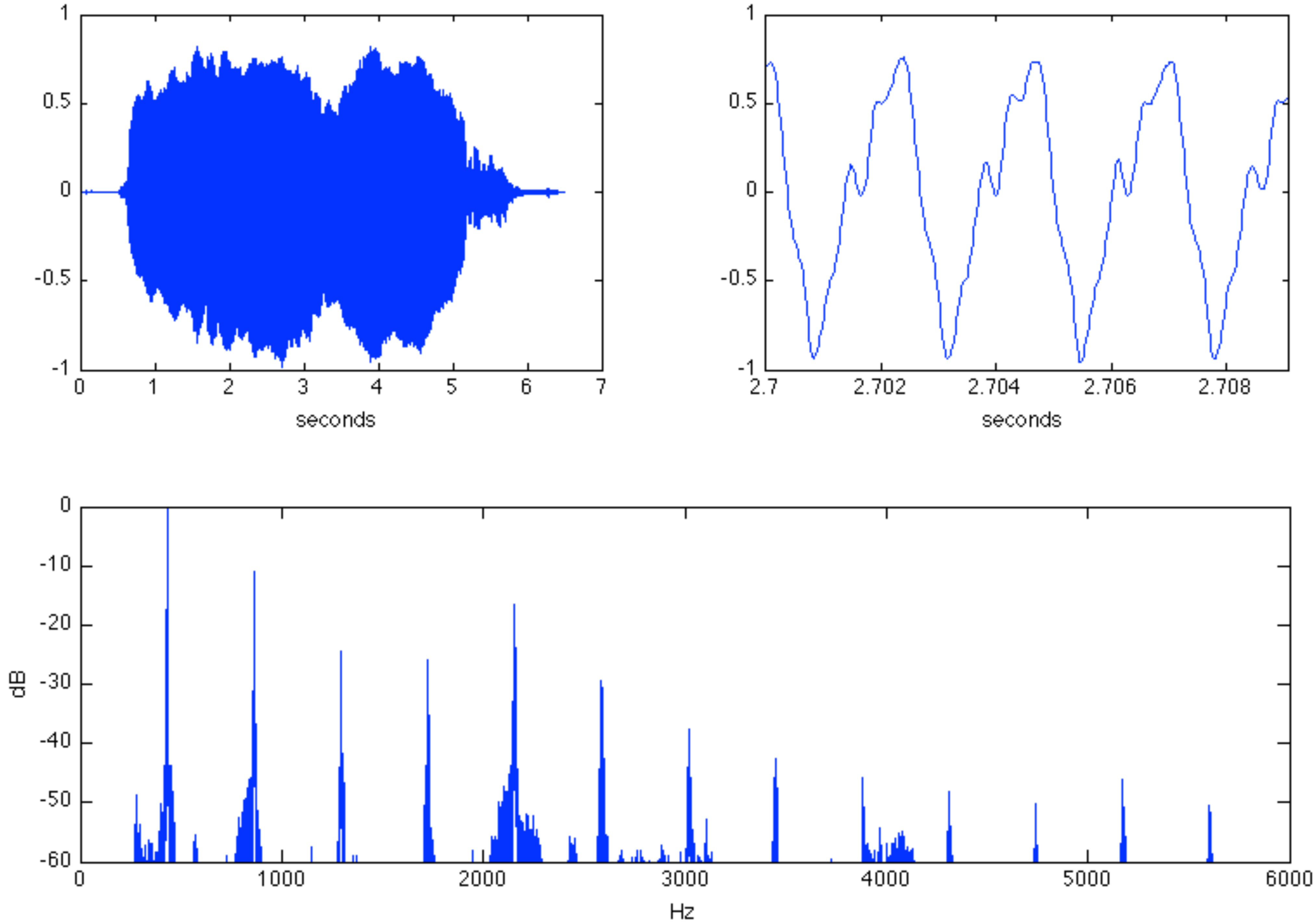
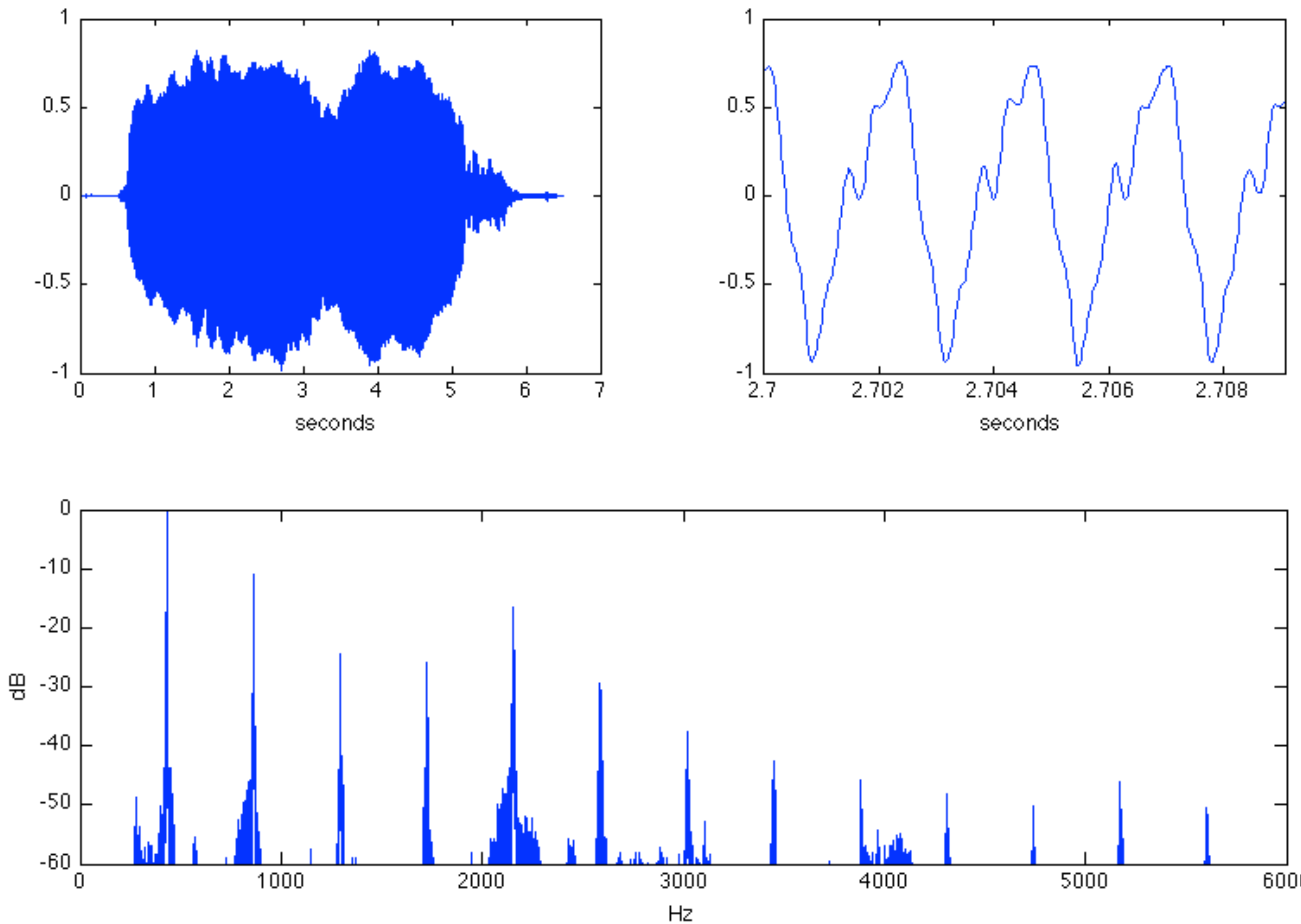
Sickly Violin (A4)



Sickly Violin (A4)



What is Timbre?

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- Timbre is related to several waveform characteristics :
  - Envelope (macroscopic wave shape)
  - Steady state (microscopic wave shape)
  - Energy spectrum

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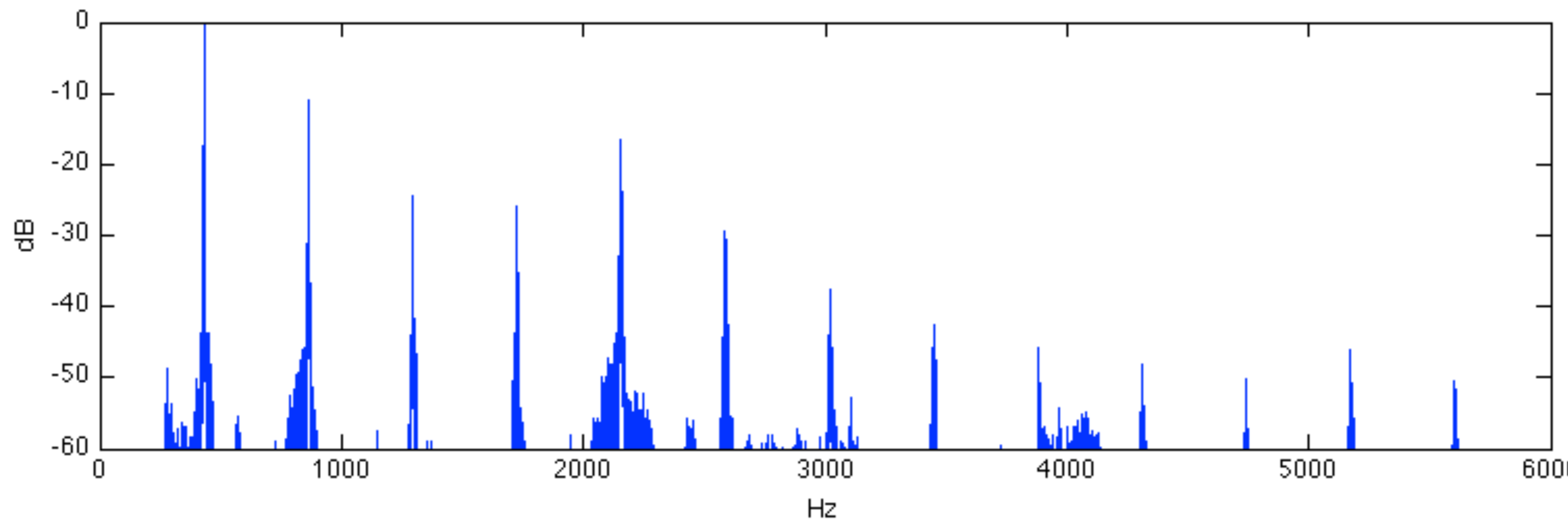

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A Question

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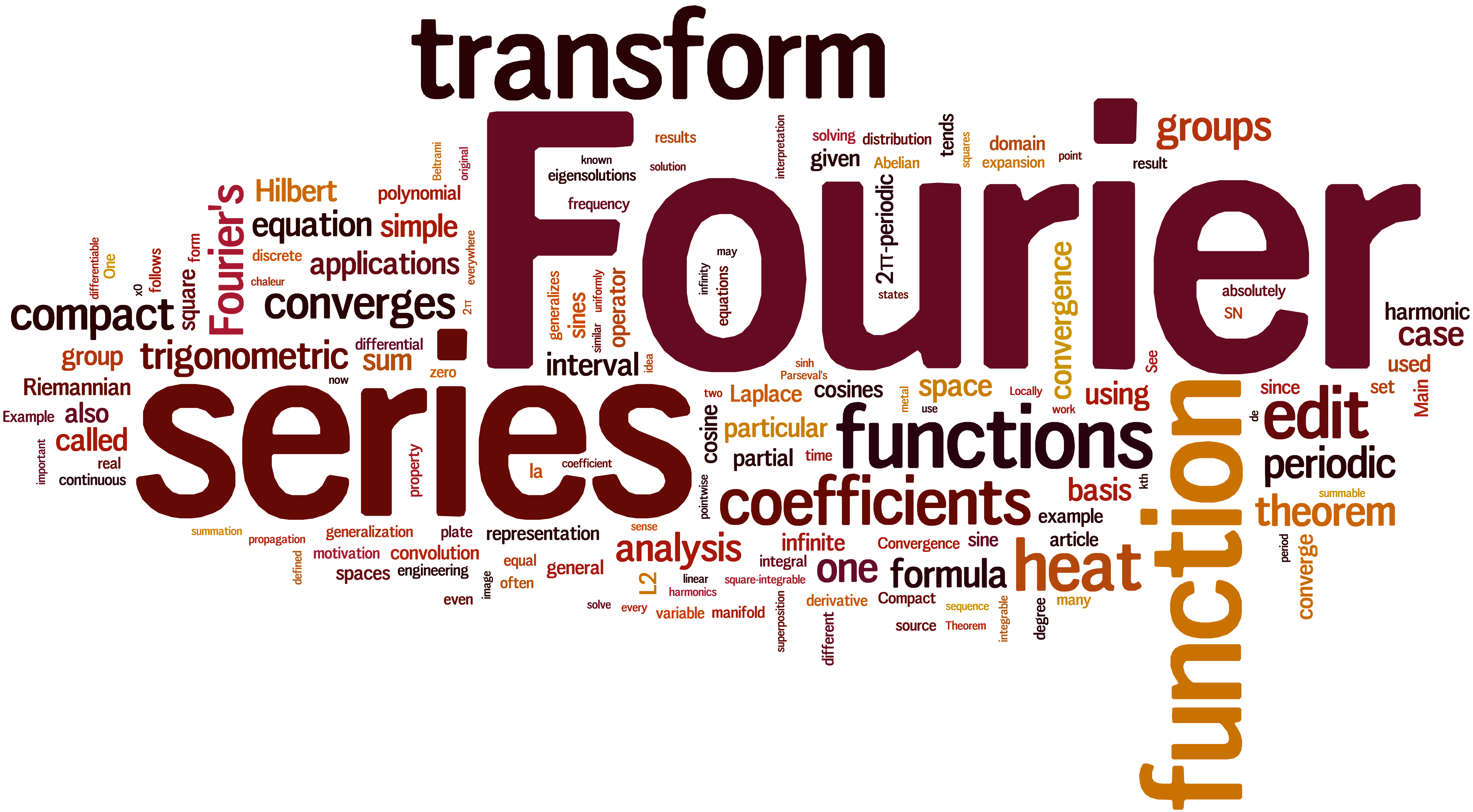
- There's a fascinating answer...

III. Fourier's Dream



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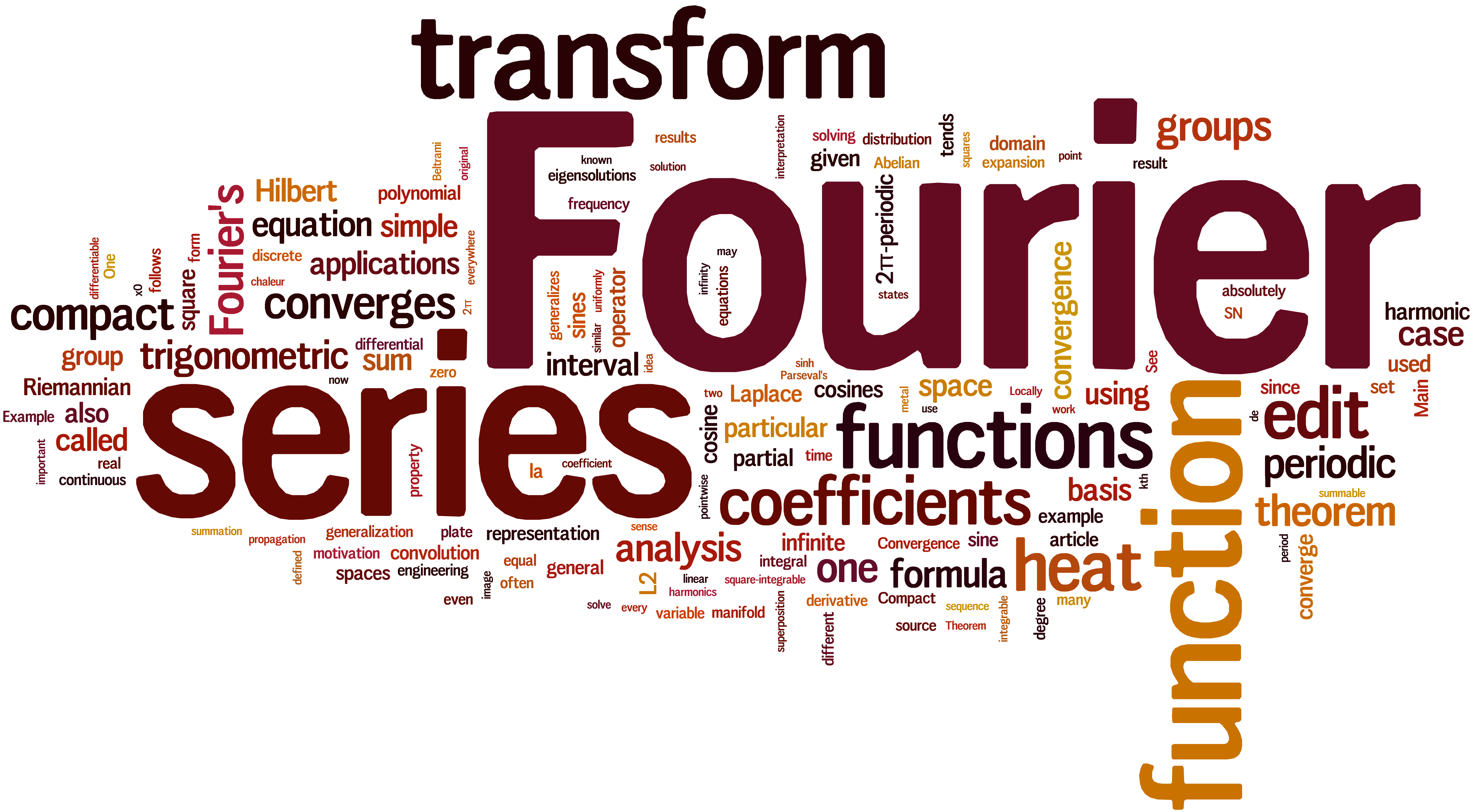

Joseph Fourier

- French, 1768 - 1830
- Commoner, orphaned at age 8
- Enthusiastic supporter of French Revolution
- Permanent Secretary of French Academy of Sciences (1822 - 1830)
- Dimensional analysis, Fourier series (1807), Fourier transform, Fourier law
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Fourier and Friend?

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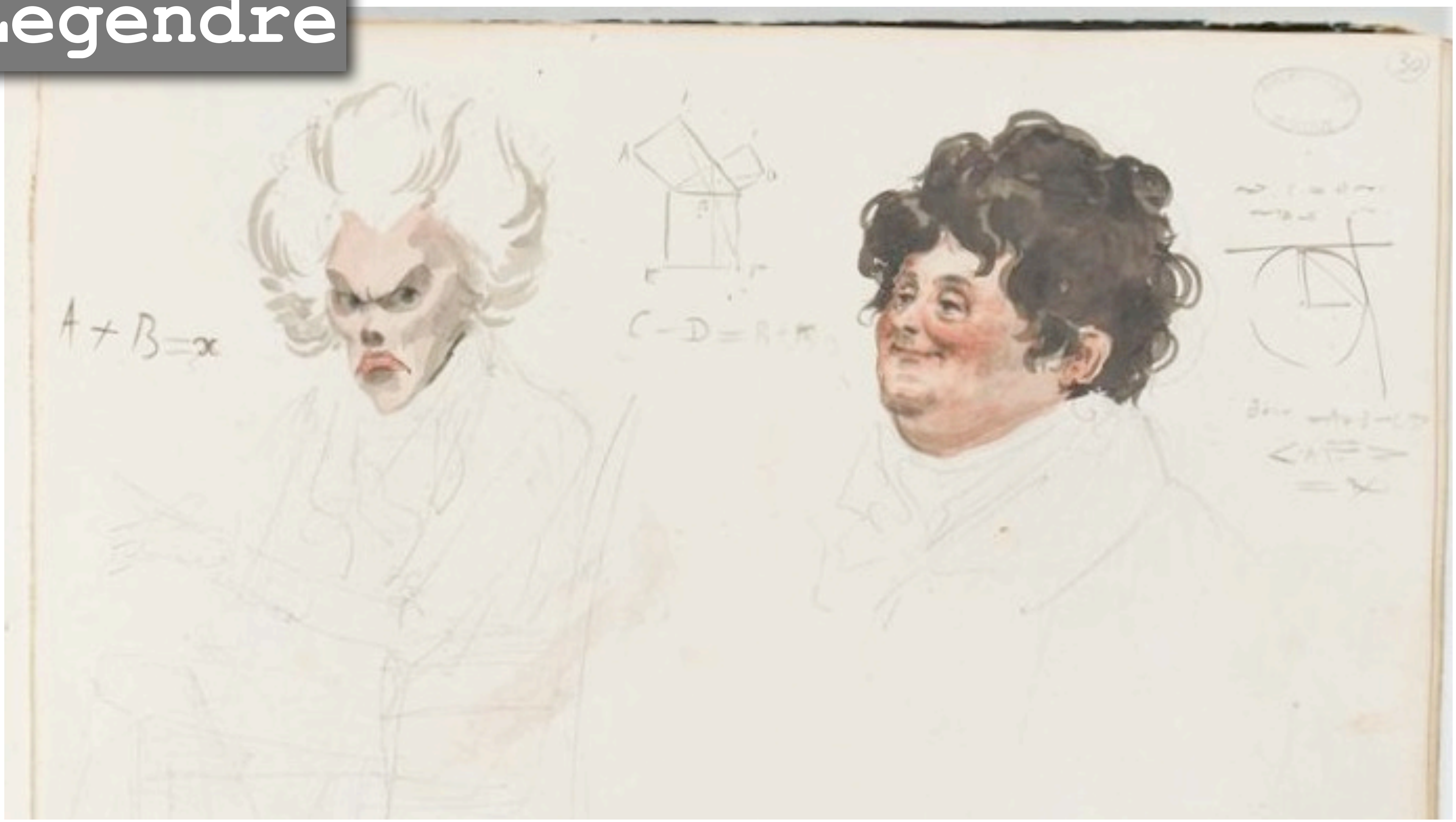
Legendre



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- What would the coefficients a_k and b_k have to be?

A Tale of Three Integrals

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$$\int_0^T \cos(2\pi k f_0 t)\ \cos(2\pi n f_0 t)\, dt = \begin{cases} \frac{T}{2} & \text{if } k = n \\ 0 & \text{if } k \neq n \end{cases}$$

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- Multiply by $\cos(2\pi n f_0 t)$ and integrate:

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- Multiply by $\cos(2\pi n f_0 t)$ and integrate:

$$\int_0^T s(t)\cos(2\pi n f_0 t)\,dt = \sum_{k=1}^{\infty} a_k \int_0^T \cos\left(2\pi k f_0 t\right)\cos\left(2\pi n f_0 t\right)\,dt$$

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$0 \;\text{ if } k \neq n$

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Fourier's Bold Idea

- Now pay heed to the Tale of Three Integrals:

$$\int_0^T s(t)\cos(2\pi n f_0 t)\,dt = a_n\,\frac{T}{2}$$

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$$a_n = \frac{2}{T} \int_0^T s(t) \cos(2\pi n f_0 t) \, dt$$

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Fourier's Theorem (1807)

- If a function s defined on $[0,T]$ can be written as

$$s(t) = \sum_{k=1}^{\infty} a_k \cos\left(2\pi k f_0 t\right) + b_k \sin\left(2\pi k f_0 t\right)$$

then

$$a_n = \frac{2}{T} \int_0^T s(t) \cos(2\pi n f_0 t)\, dt$$

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- More generally , which s can be written as

$$s(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos\left(2\pi k f_0 t\right) + b_k \sin\left(2\pi k f_0 t\right)$$

Fourier's (Overly) Bold Answer

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Every function!

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~~Every function!~~

A Question of Convergence

- Partial sums of the Fourier series:

$$s_N(t) = a_0 + \sum_{k=1}^{N} a_k \cos(2\pi k f_0 t) + b_k \sin(2\pi k f_0 t)$$

- For which s do we have

$$s_N \to s \text{ as } N \to \infty$$

(and in what sense?)

Dirichlet's Theorem (1829)

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* (don't ask)

Riesz-Fischer Theorem (1907)

A: $\int_0^T s^2(t)\,dt < \infty$

B: $\int_0^T (s(t) - s_N(t))^2\,dt \to 0$

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- Note: physically,

$$\int_0^T s^2(t)\,dt = \text{ energy of } s$$

Carleson's Theorem (1966)

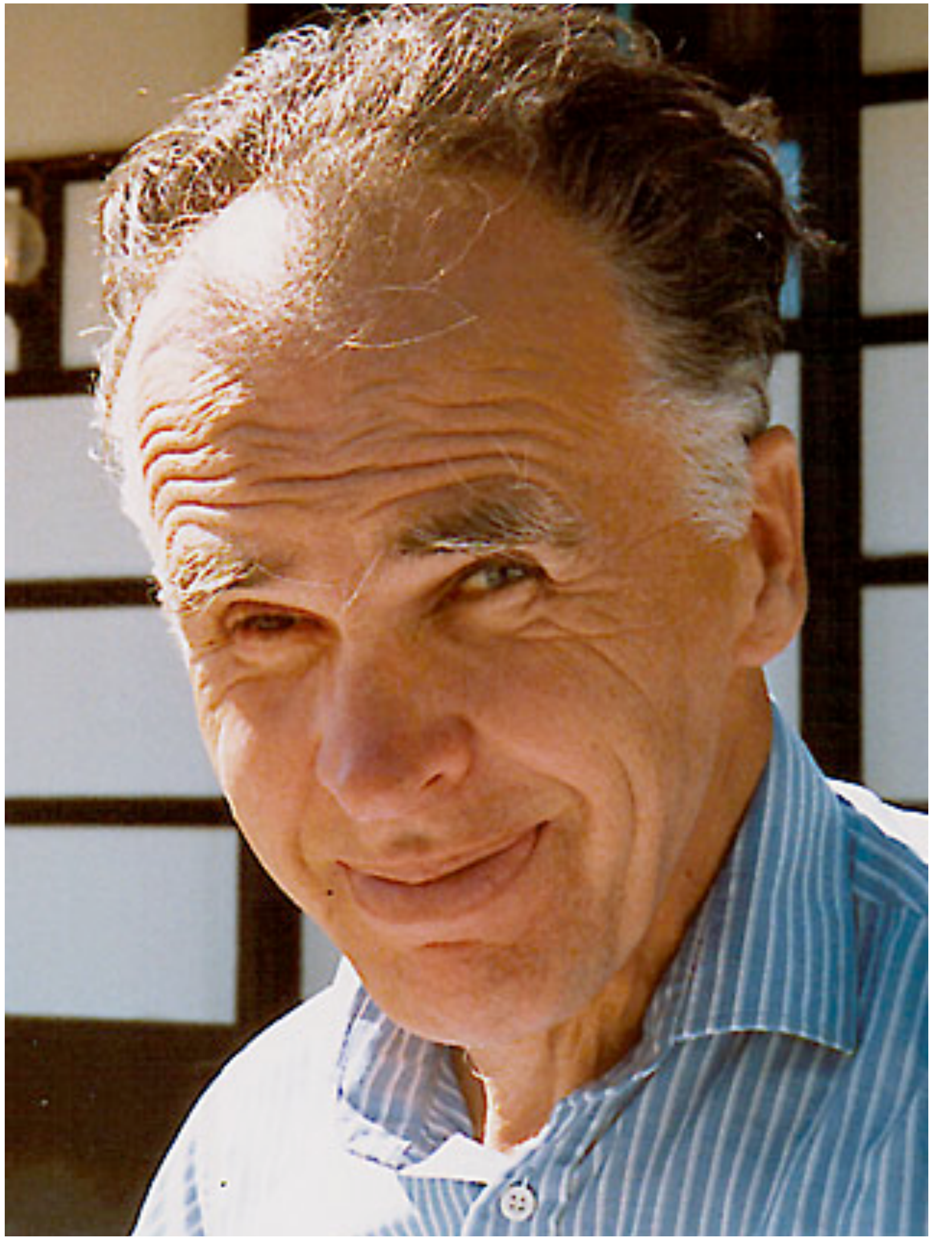
- If s is measurable, and

$$\int_0^T s^2(t)\,dt < \infty$$

then

$$\lim_{N\to\infty} s_N(t) = s(t)$$

for *almost every* t in $[0, T]$.



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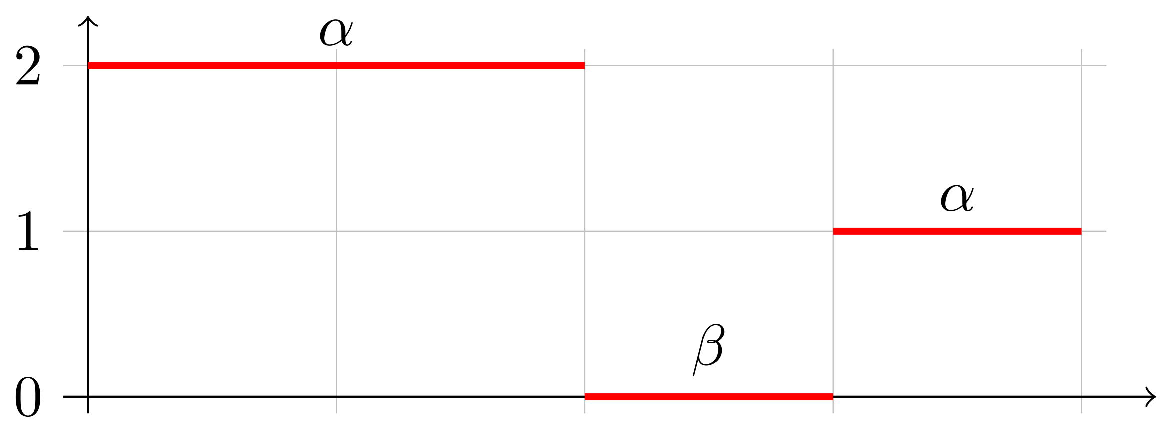
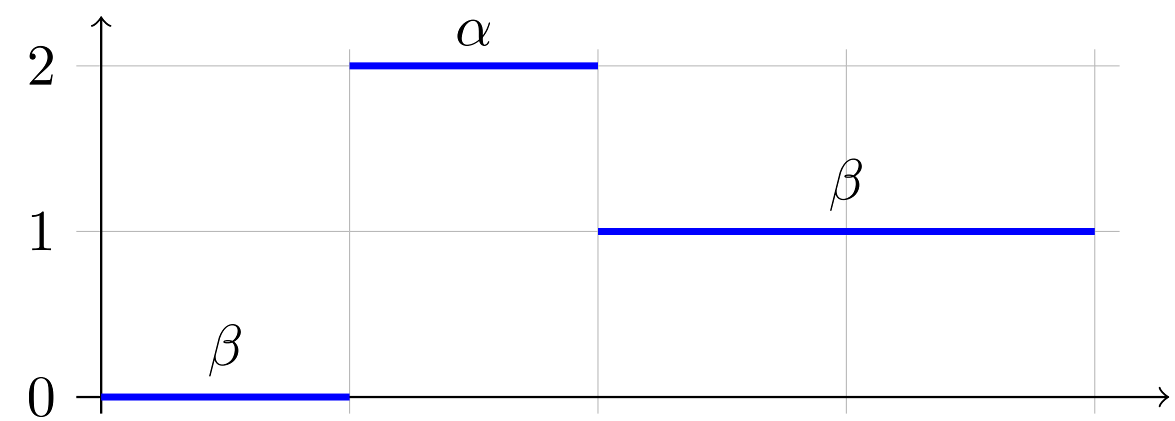
- His "bold idea" lead to a whole new field of mathematics.
- Spurred mathematicians to think more rigorously about the "function" concept.
- His approach continues to inspire new mathematics
  - Wavelets
  - Empirical mode decomposition

Opus 2: Fourier's Triumph

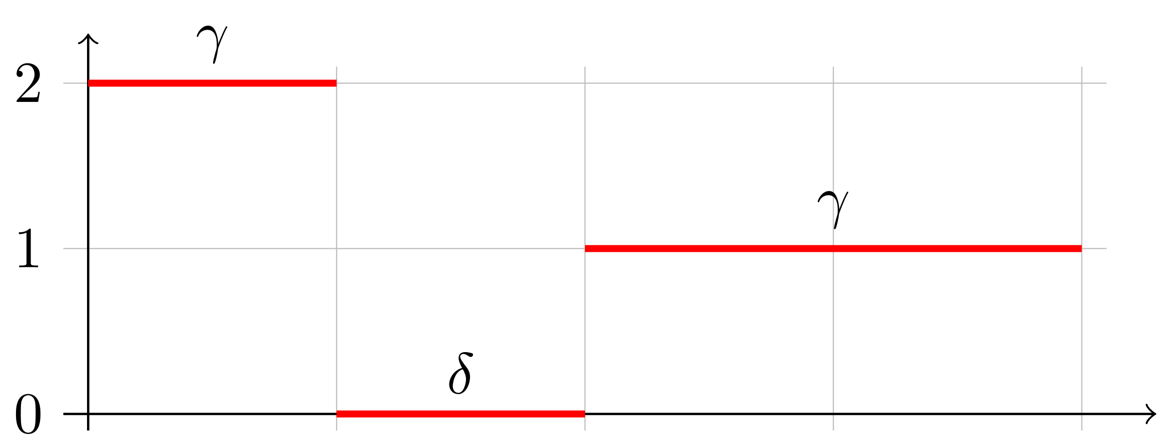
Pattern α



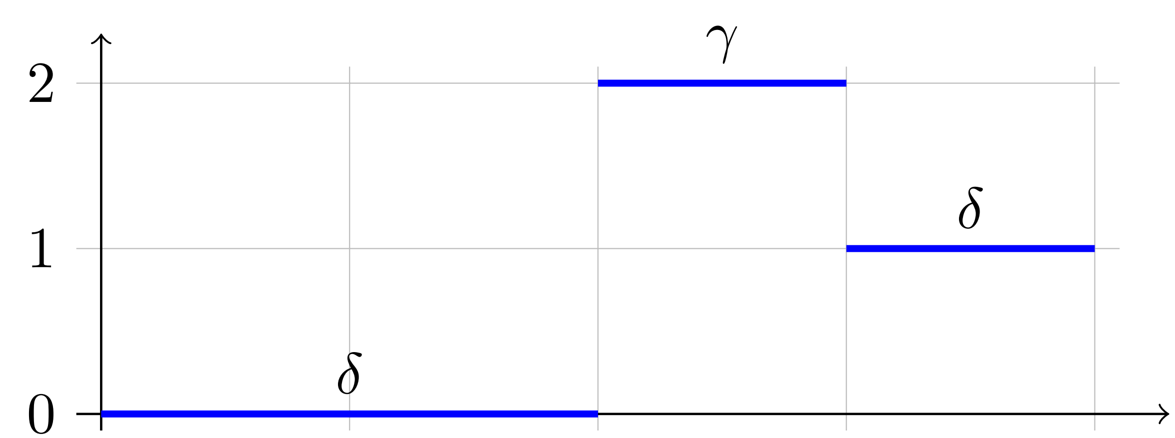
Pattern β



Pattern γ

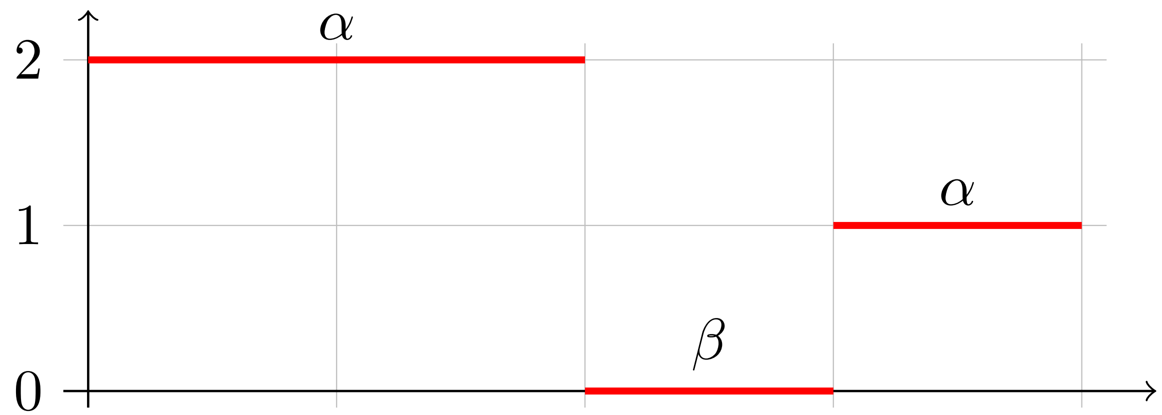


Pattern δ

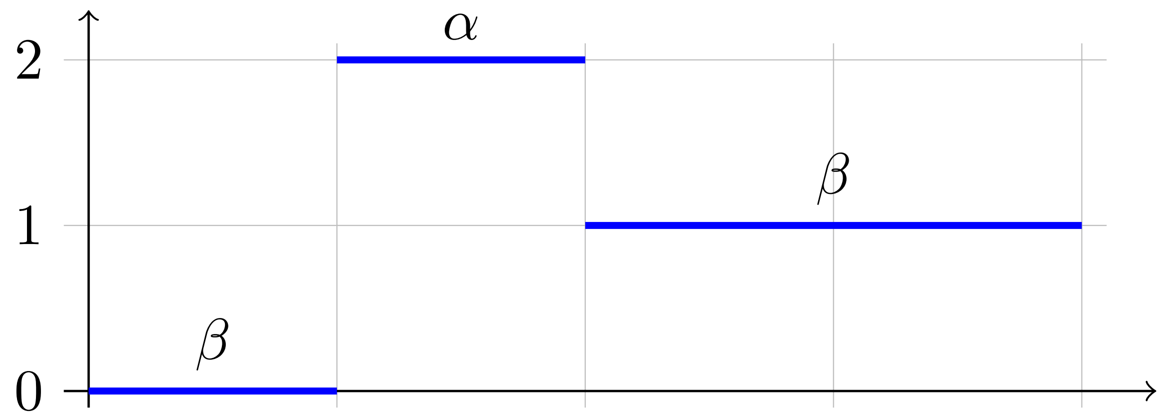


Opus 2: Fourier's Triumph

Pattern α



Pattern β



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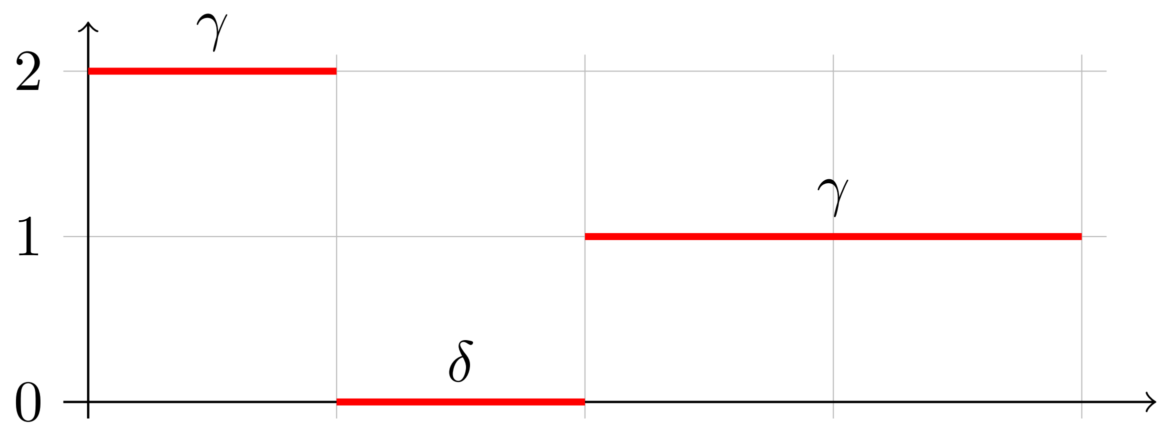

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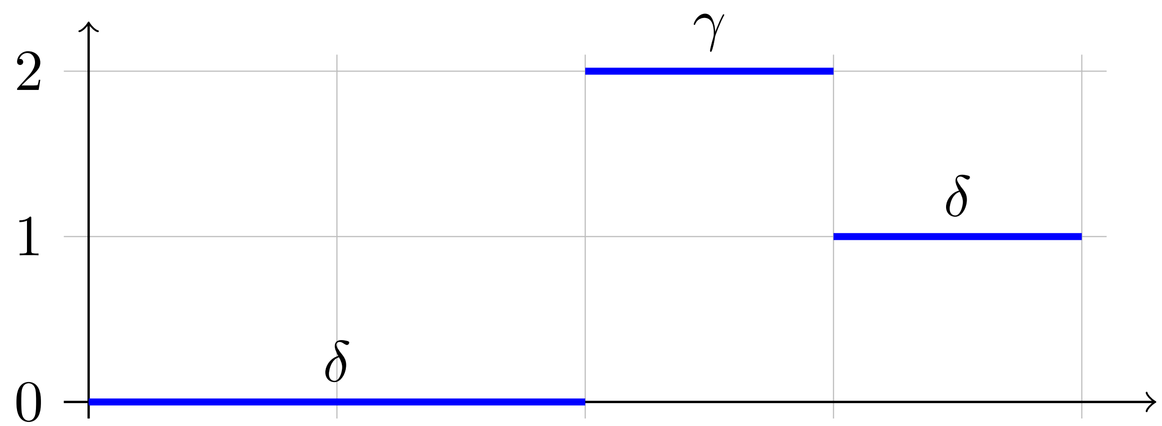

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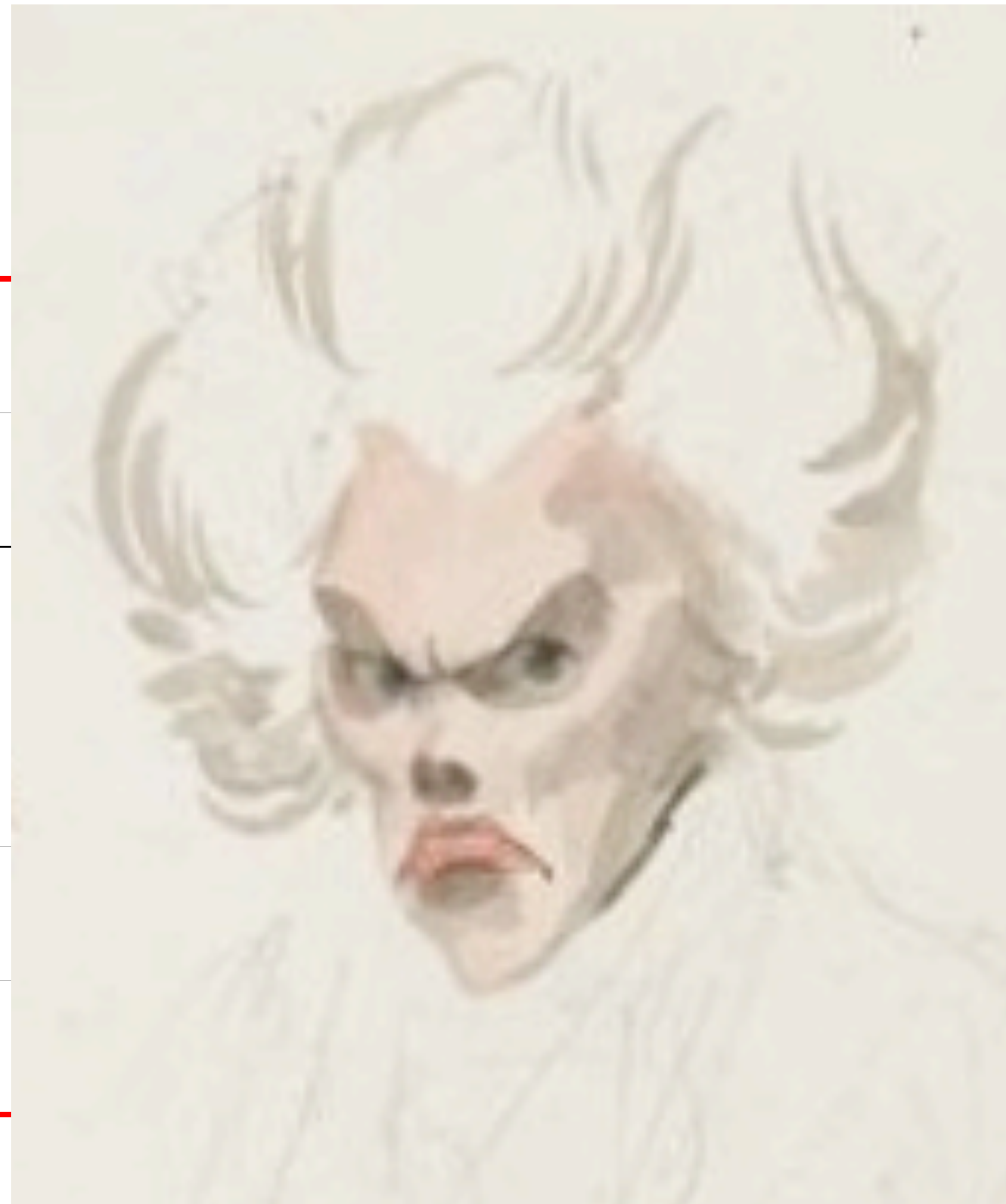

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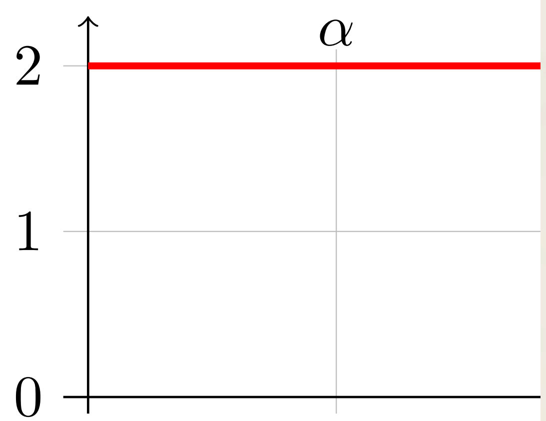

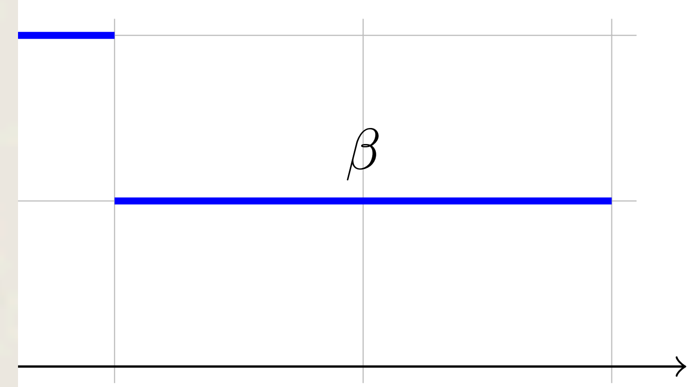

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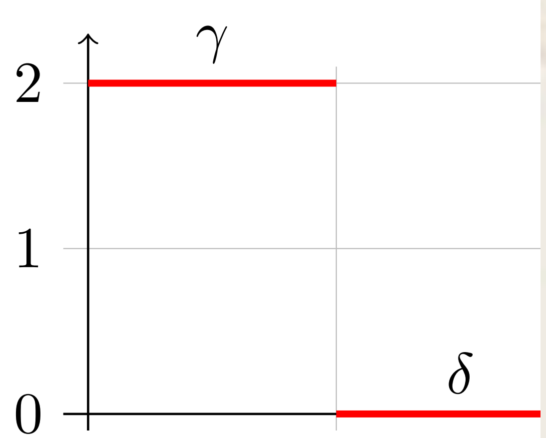
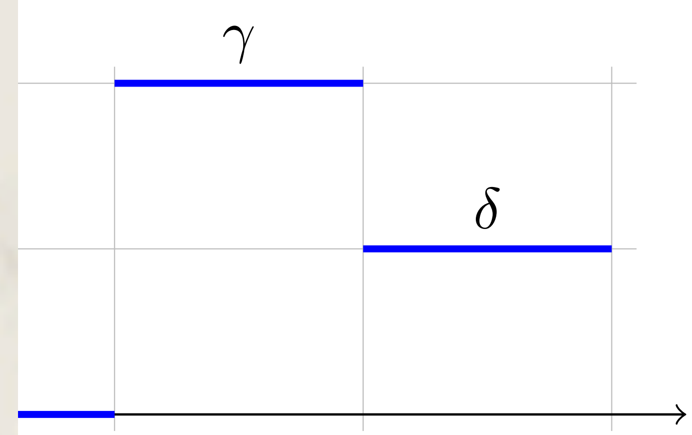
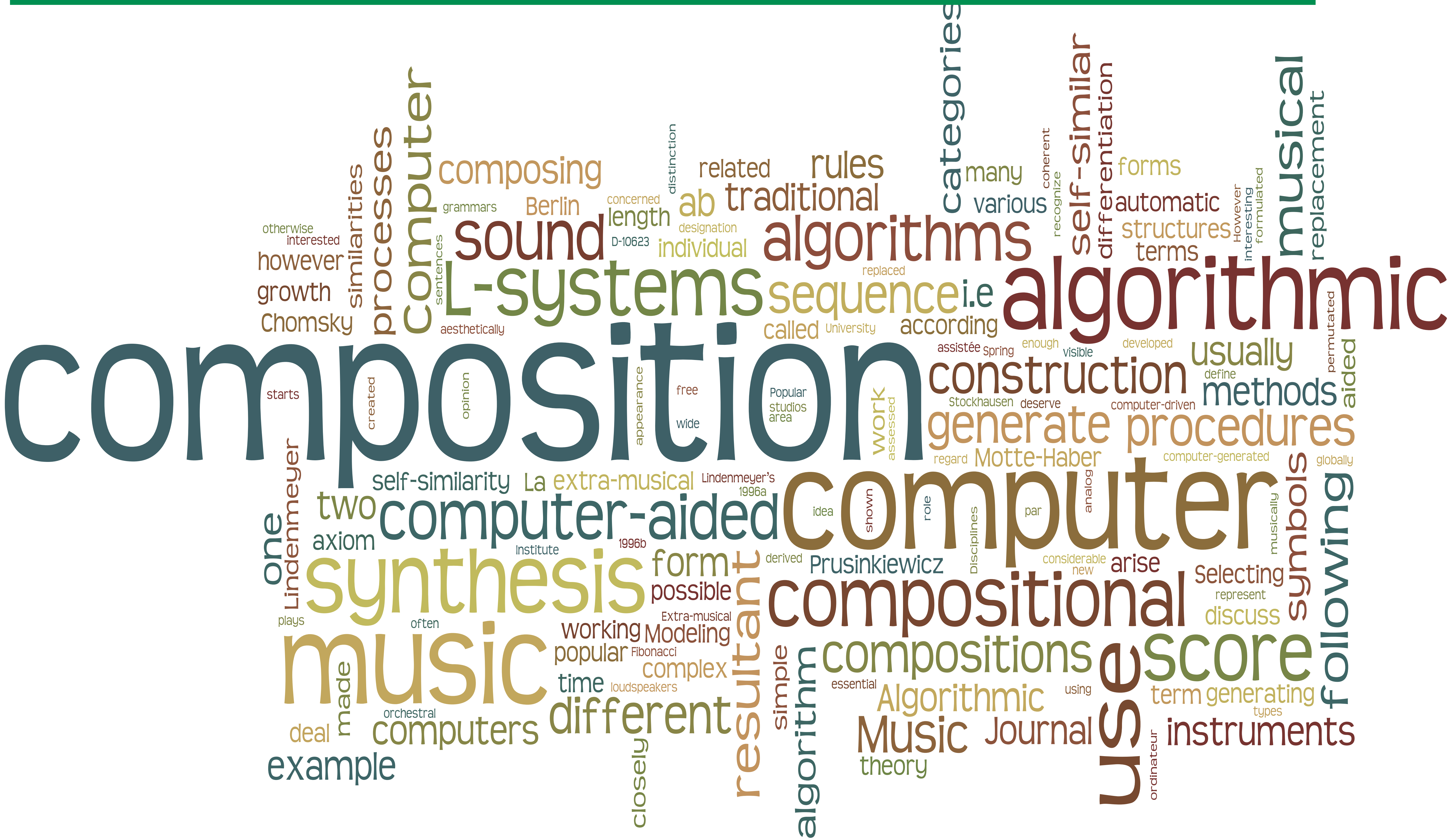
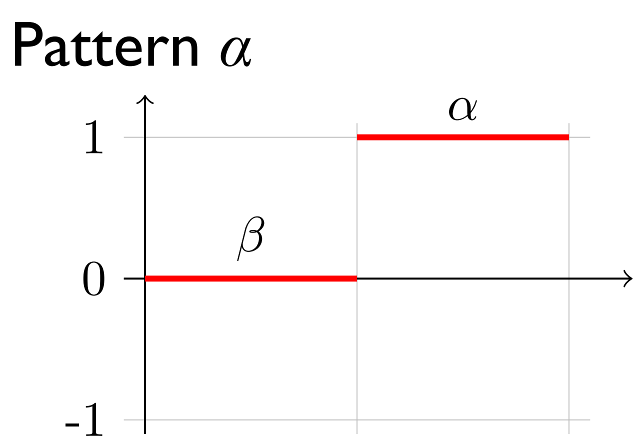


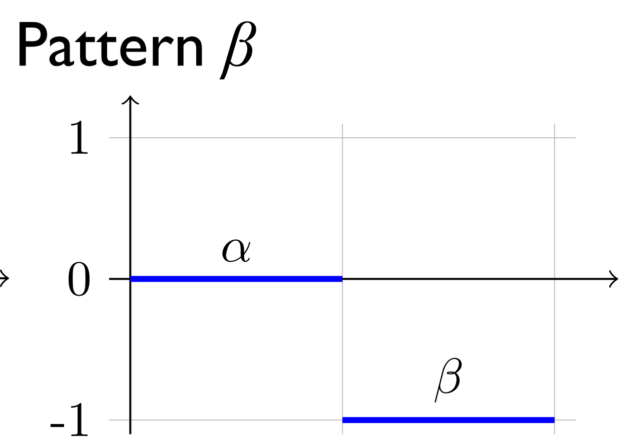


IV. The Music of Mathematics



A Musical Fractal





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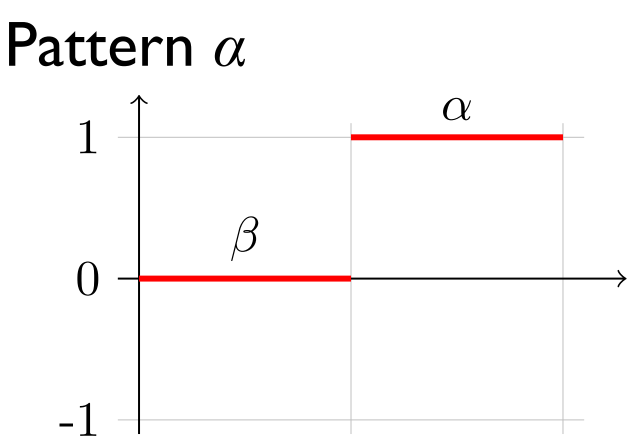

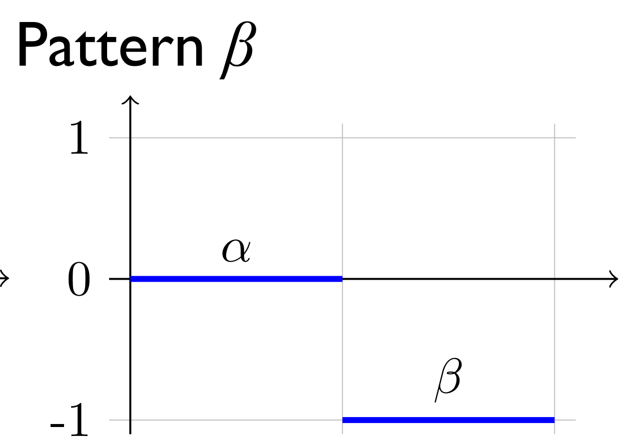

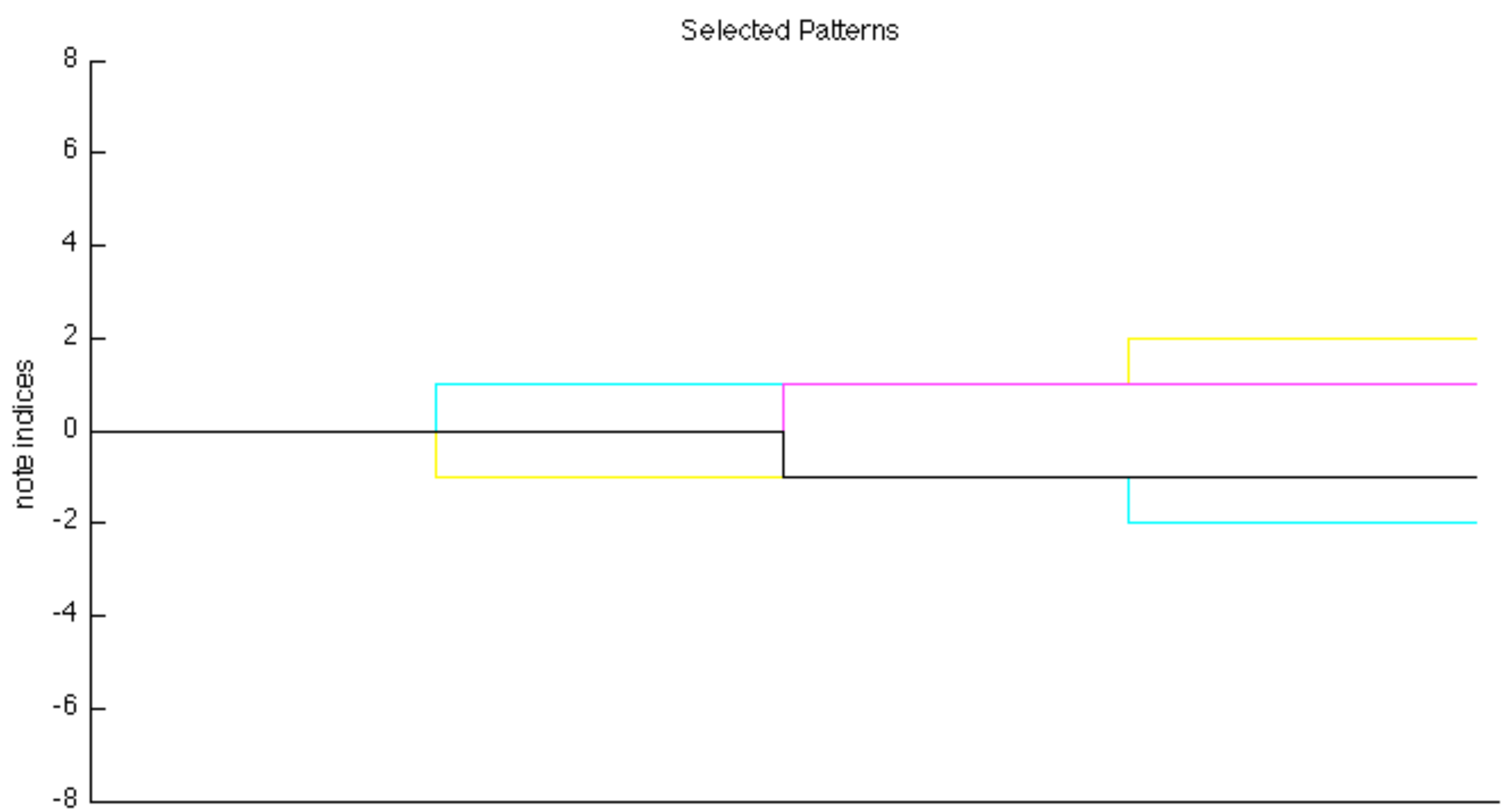

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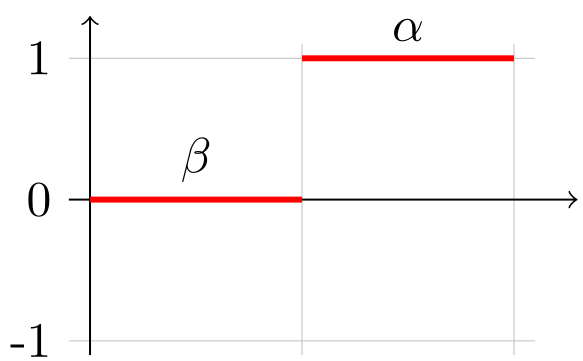
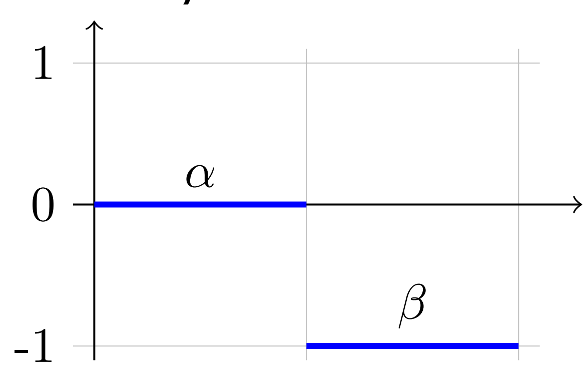
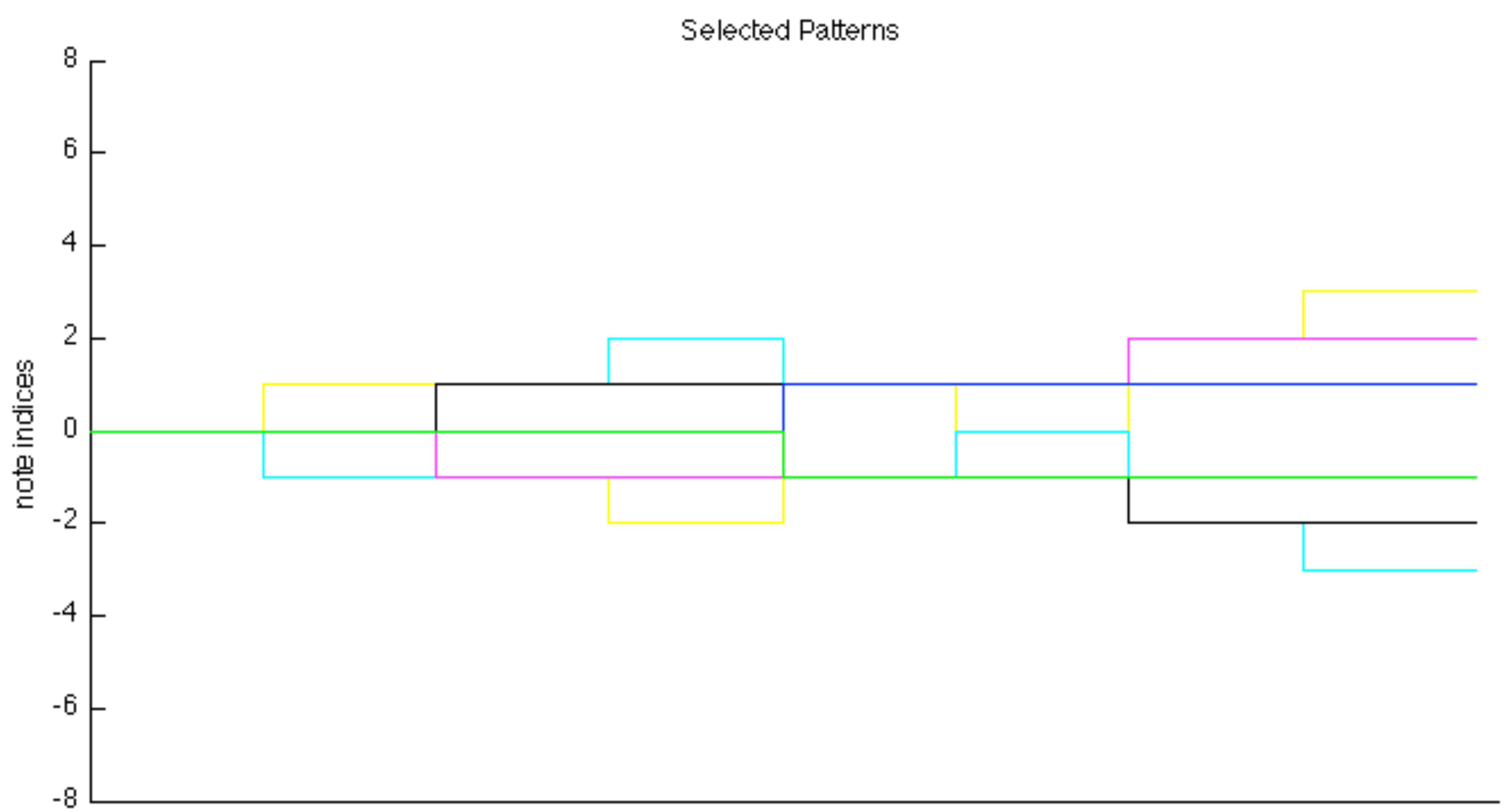
Pattern α



Pattern β





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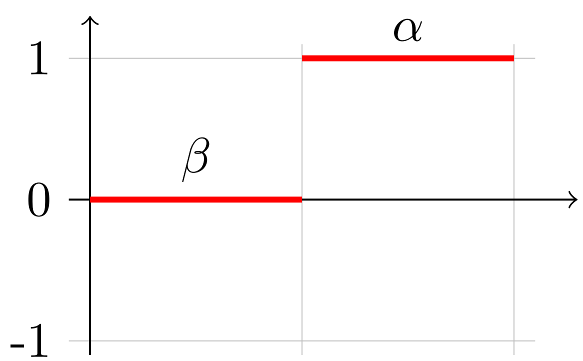

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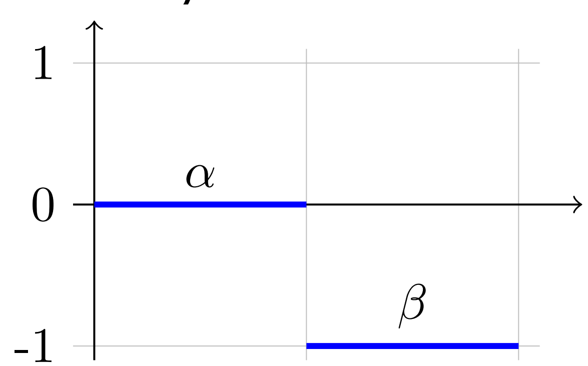

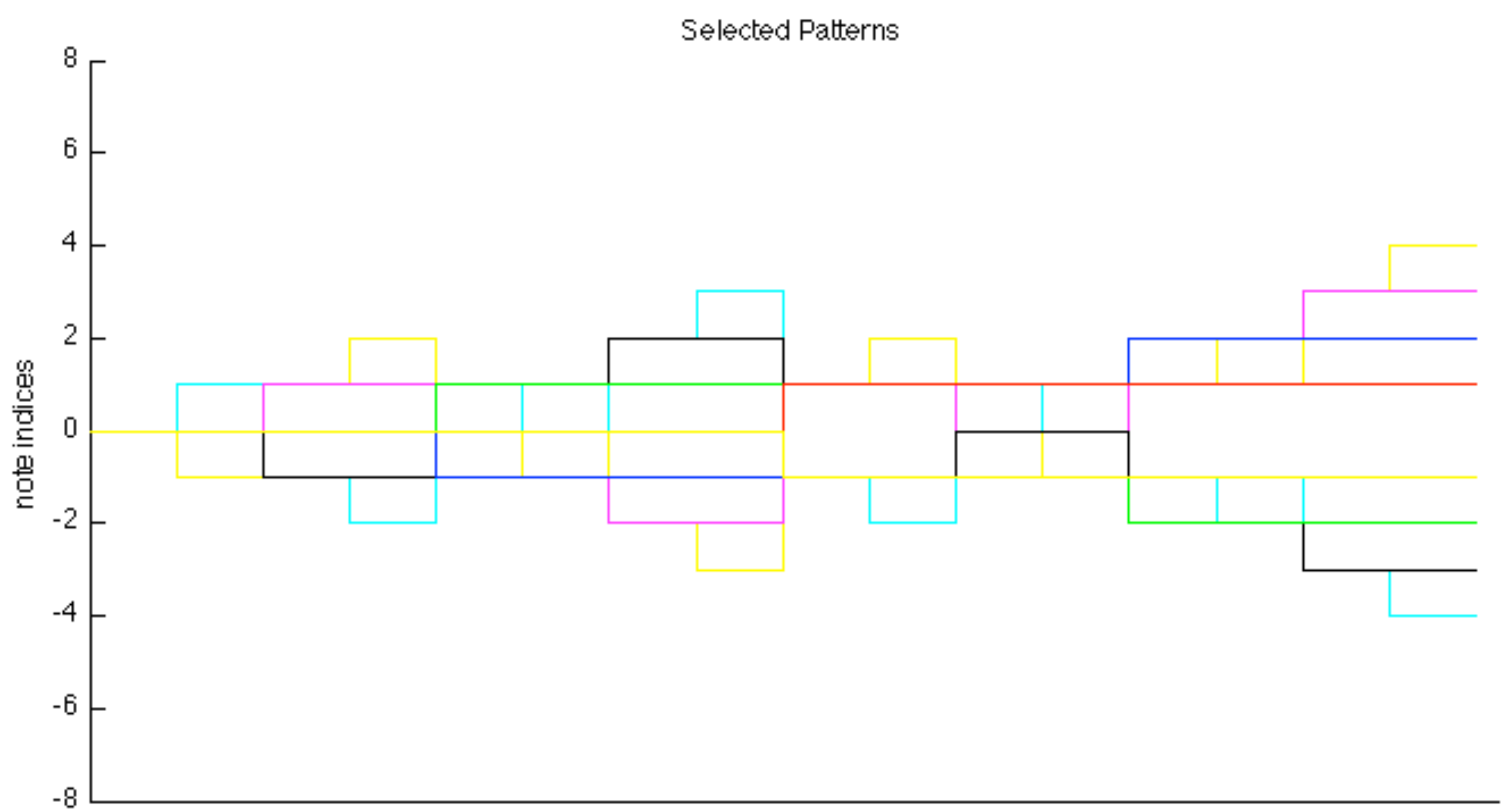

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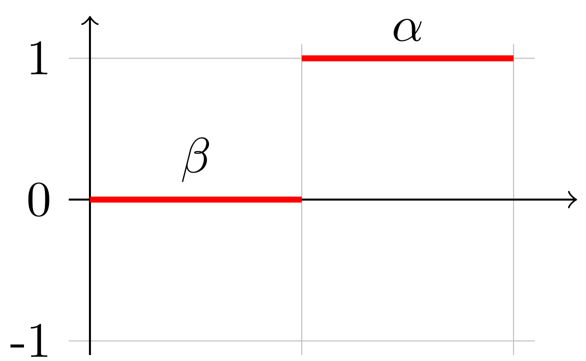

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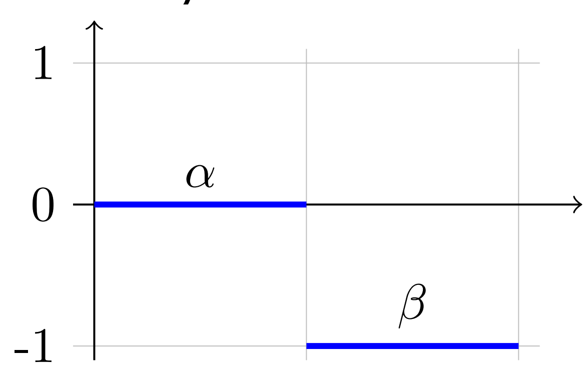
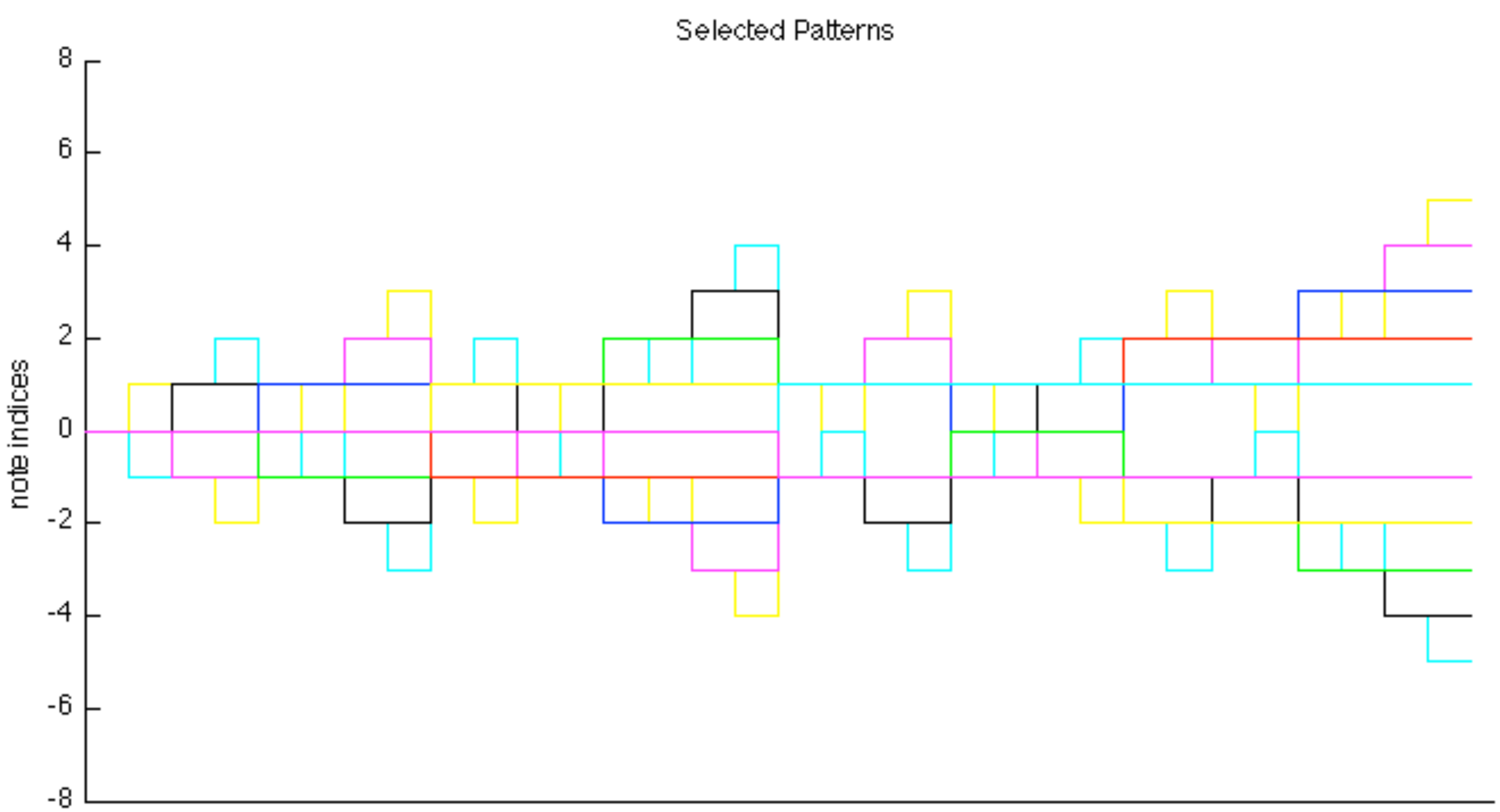
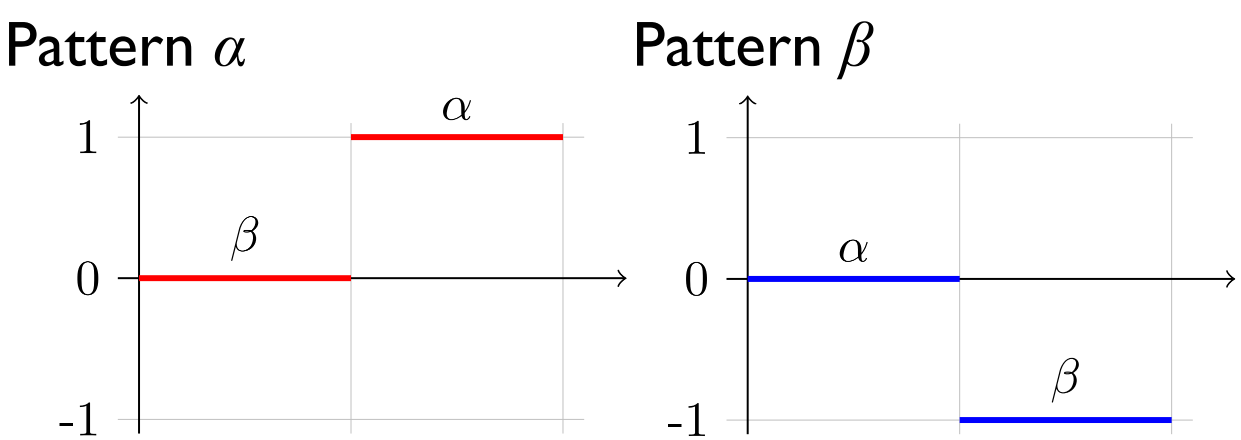
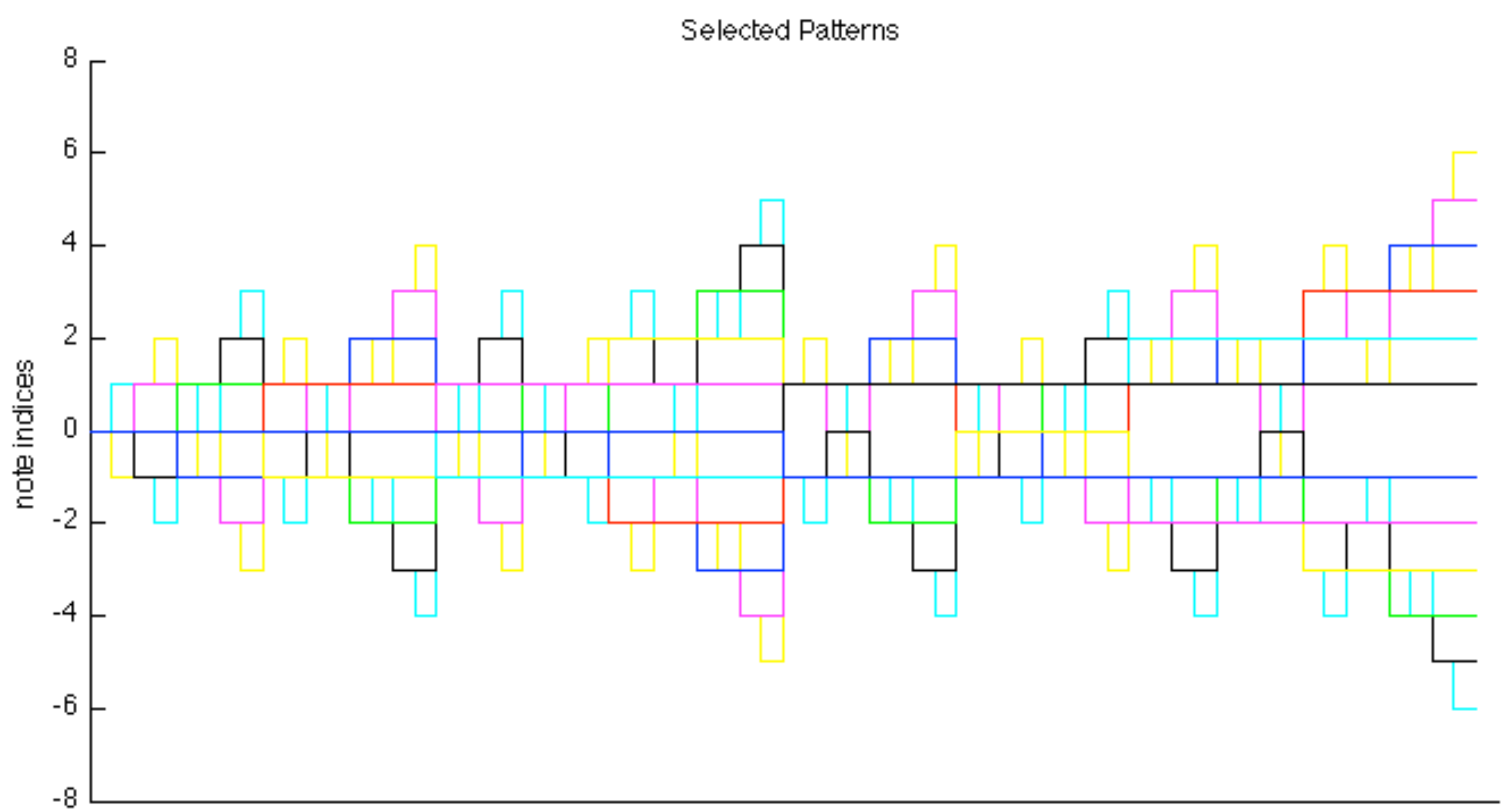


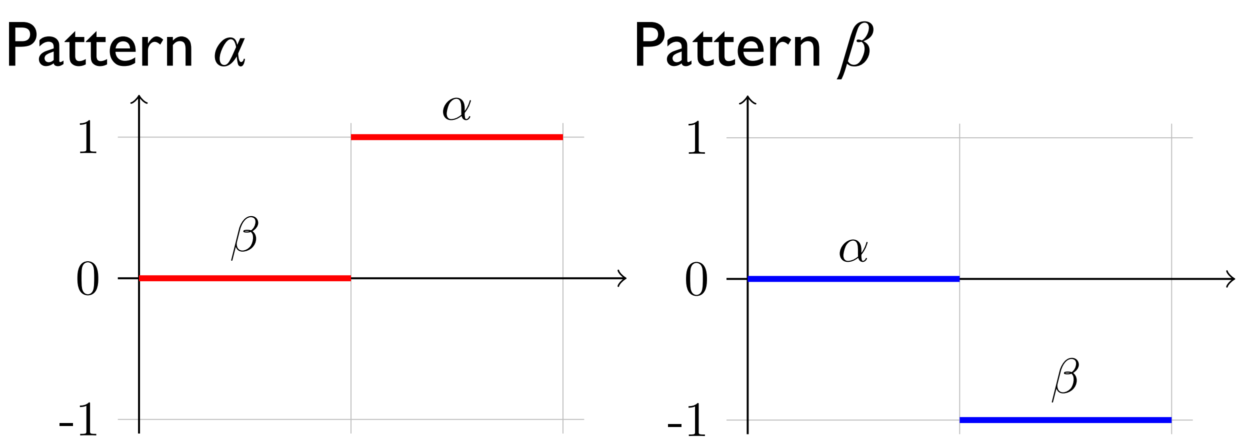


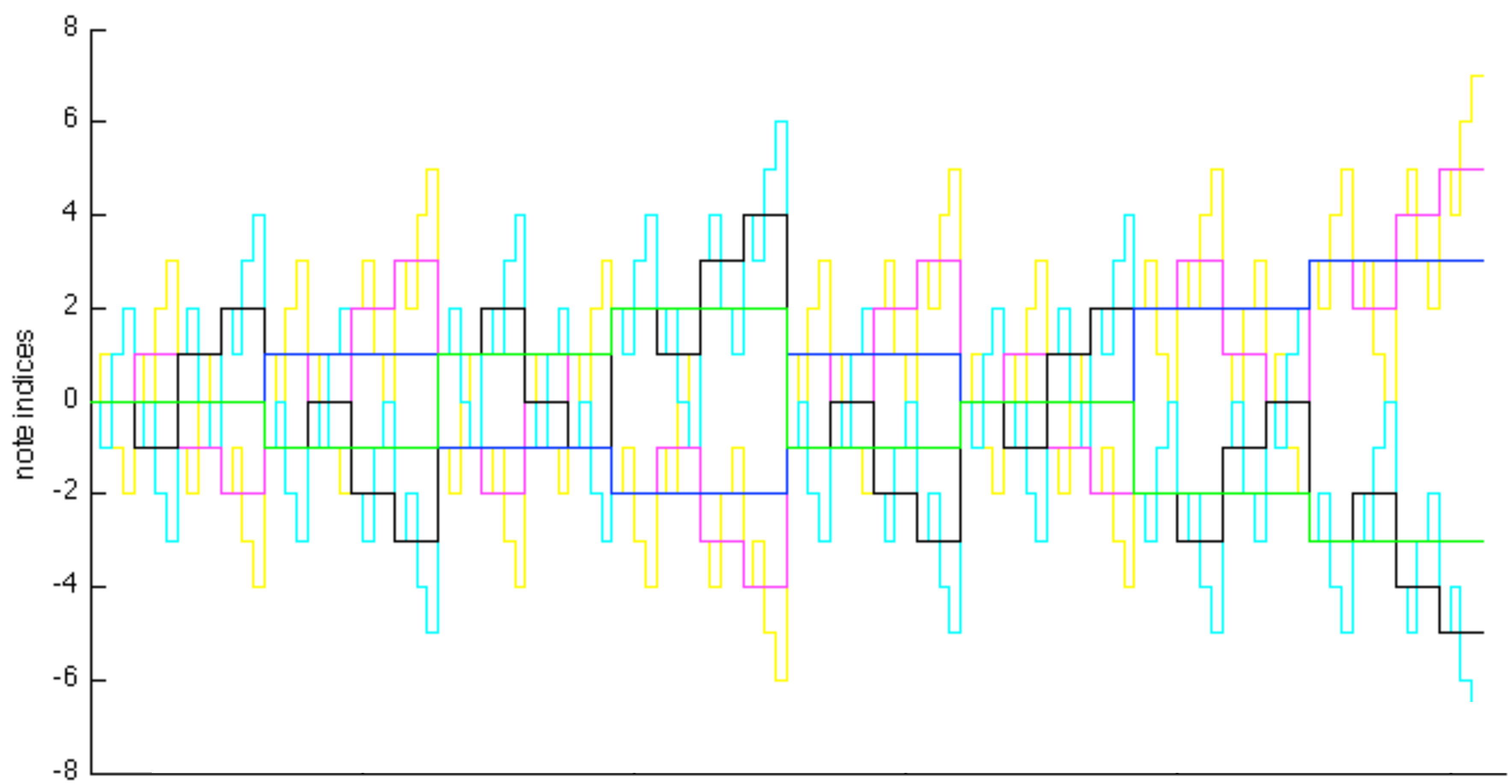
A Musical Fractal





A Musical Fractal





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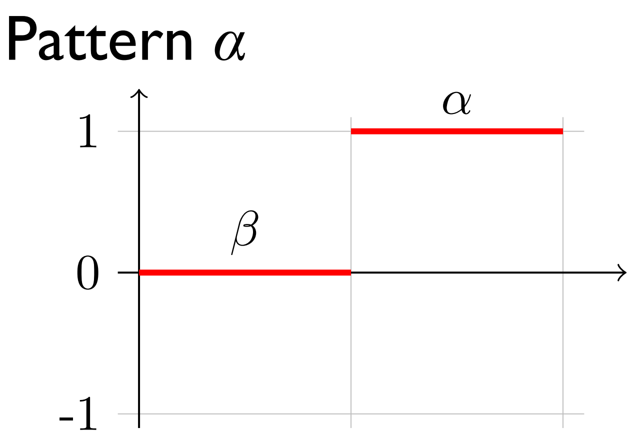

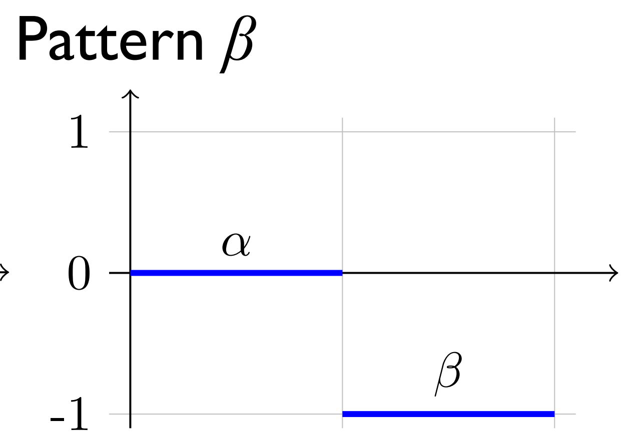

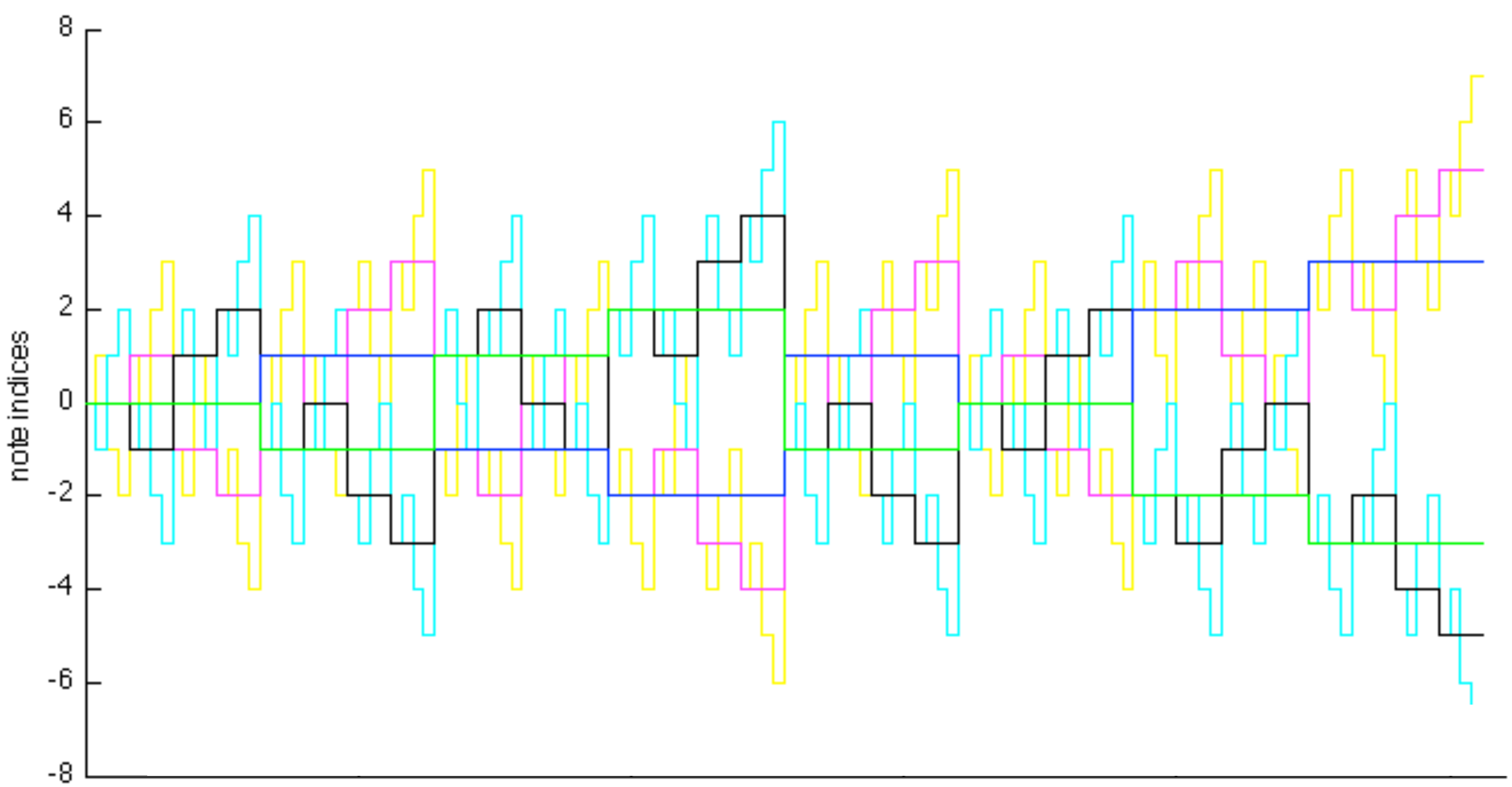

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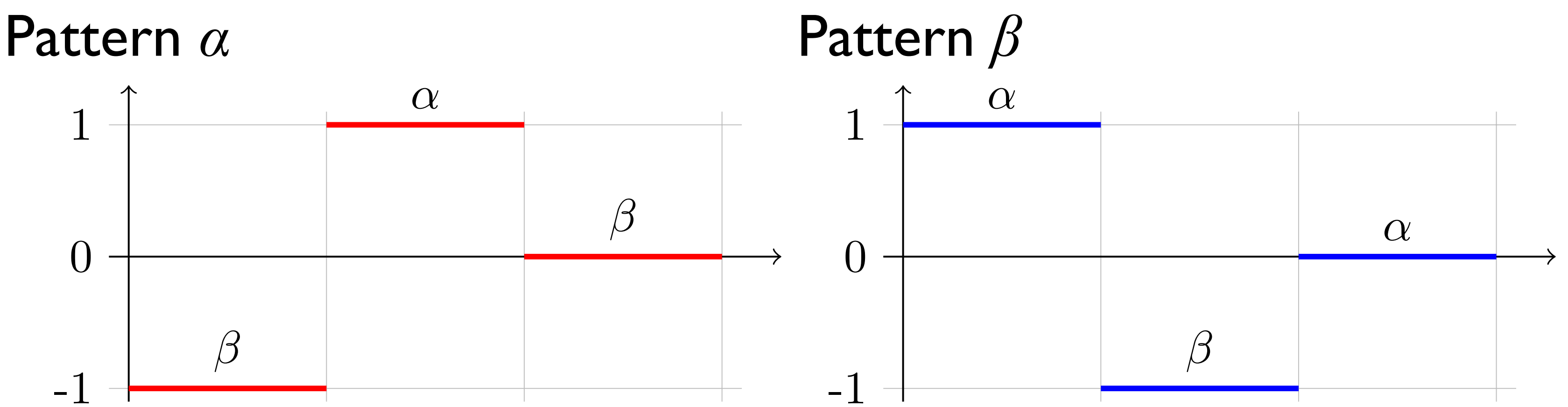
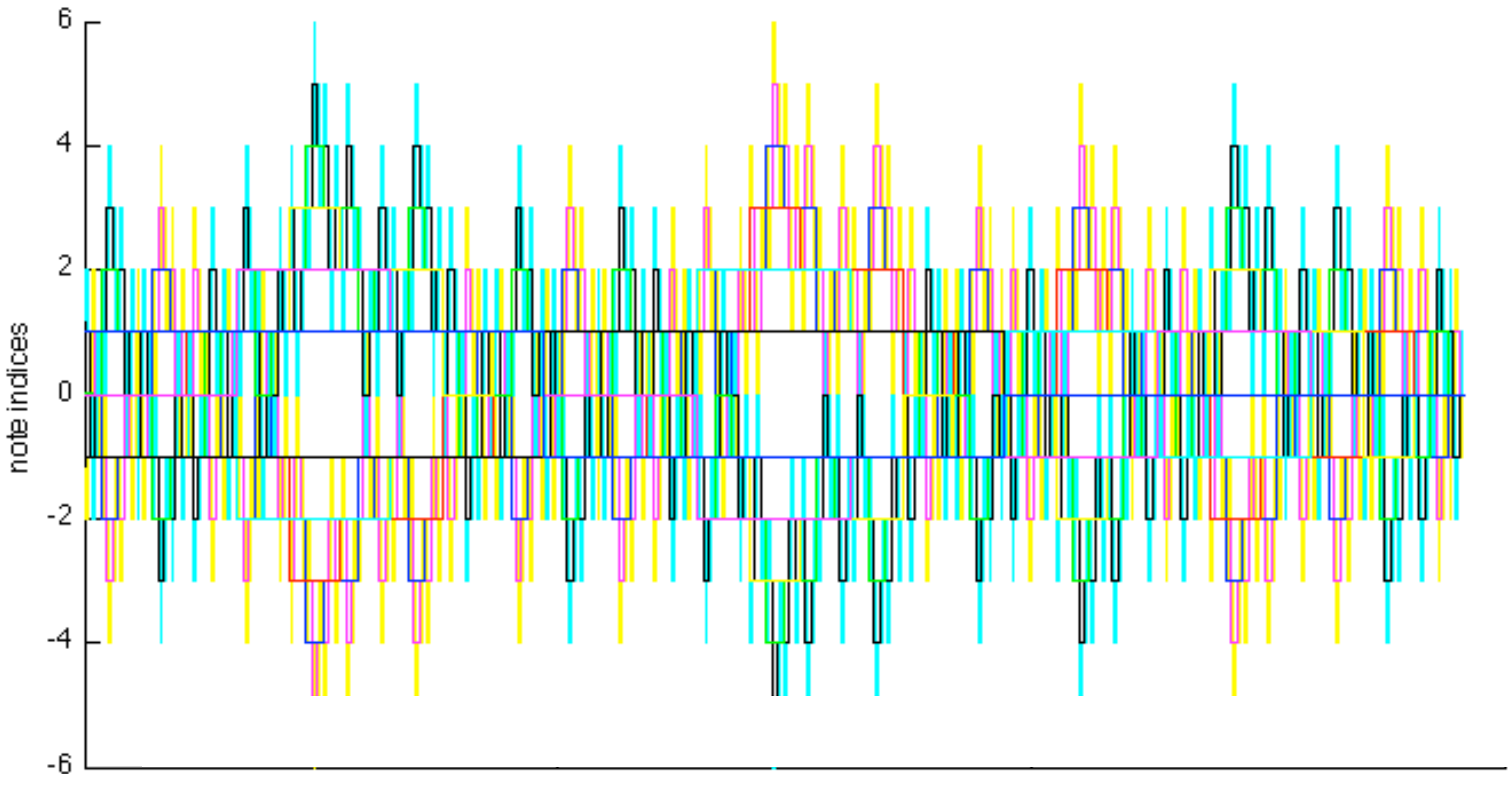
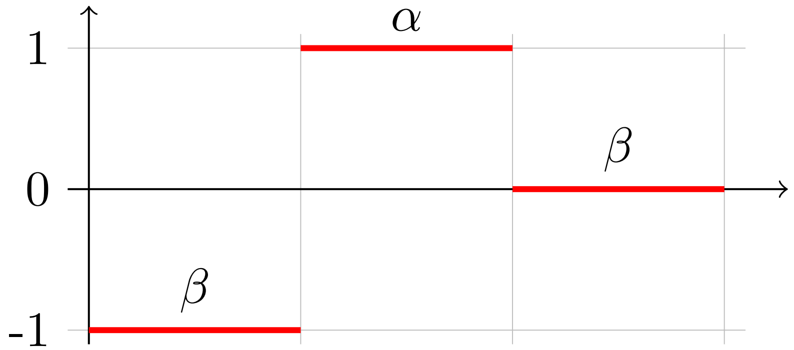


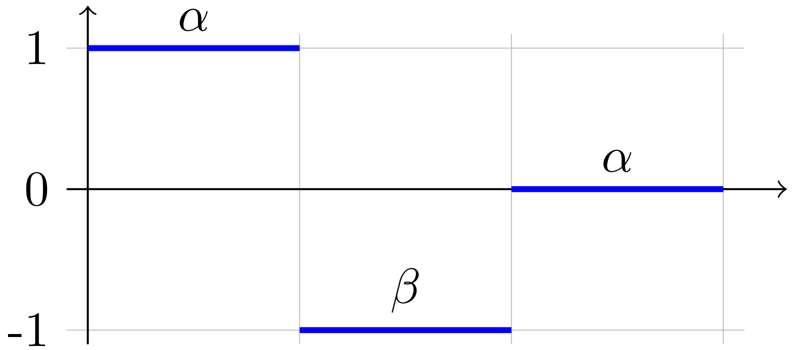


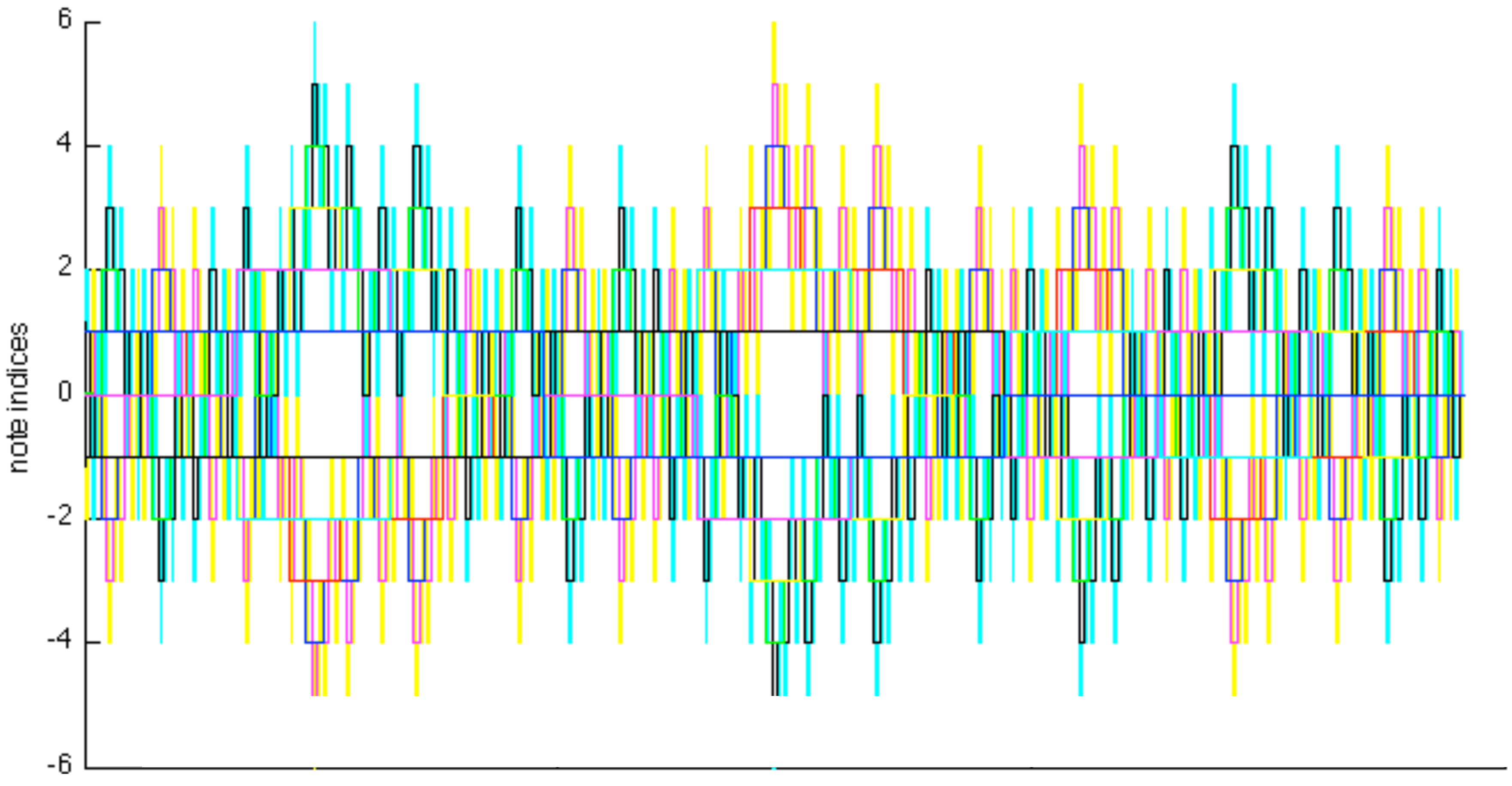
Another Musical Fractal

Pattern α



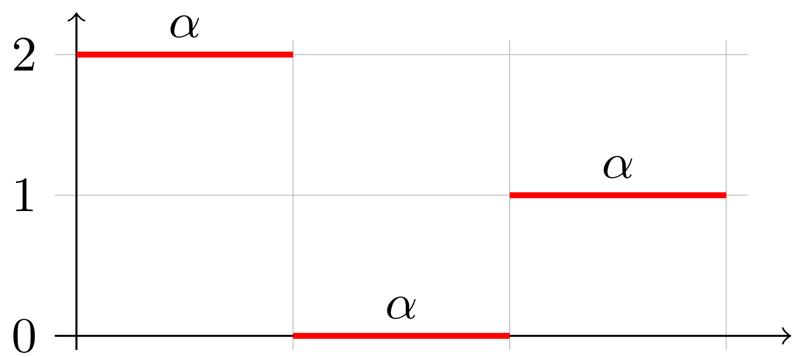
Pattern β

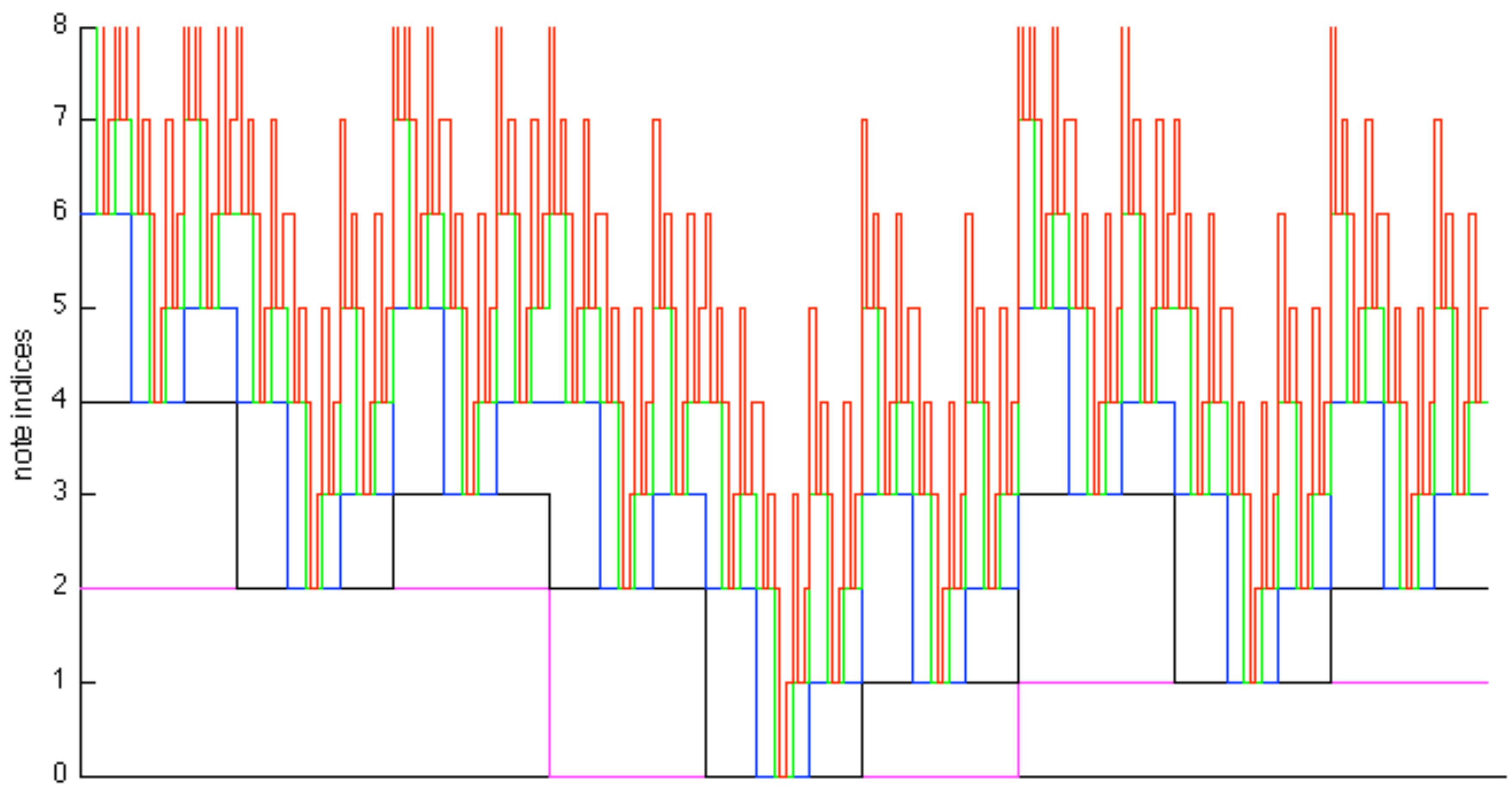




And Another...

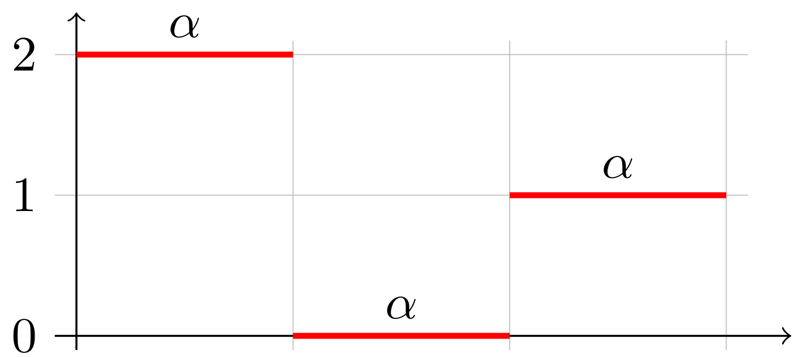
Pattern α

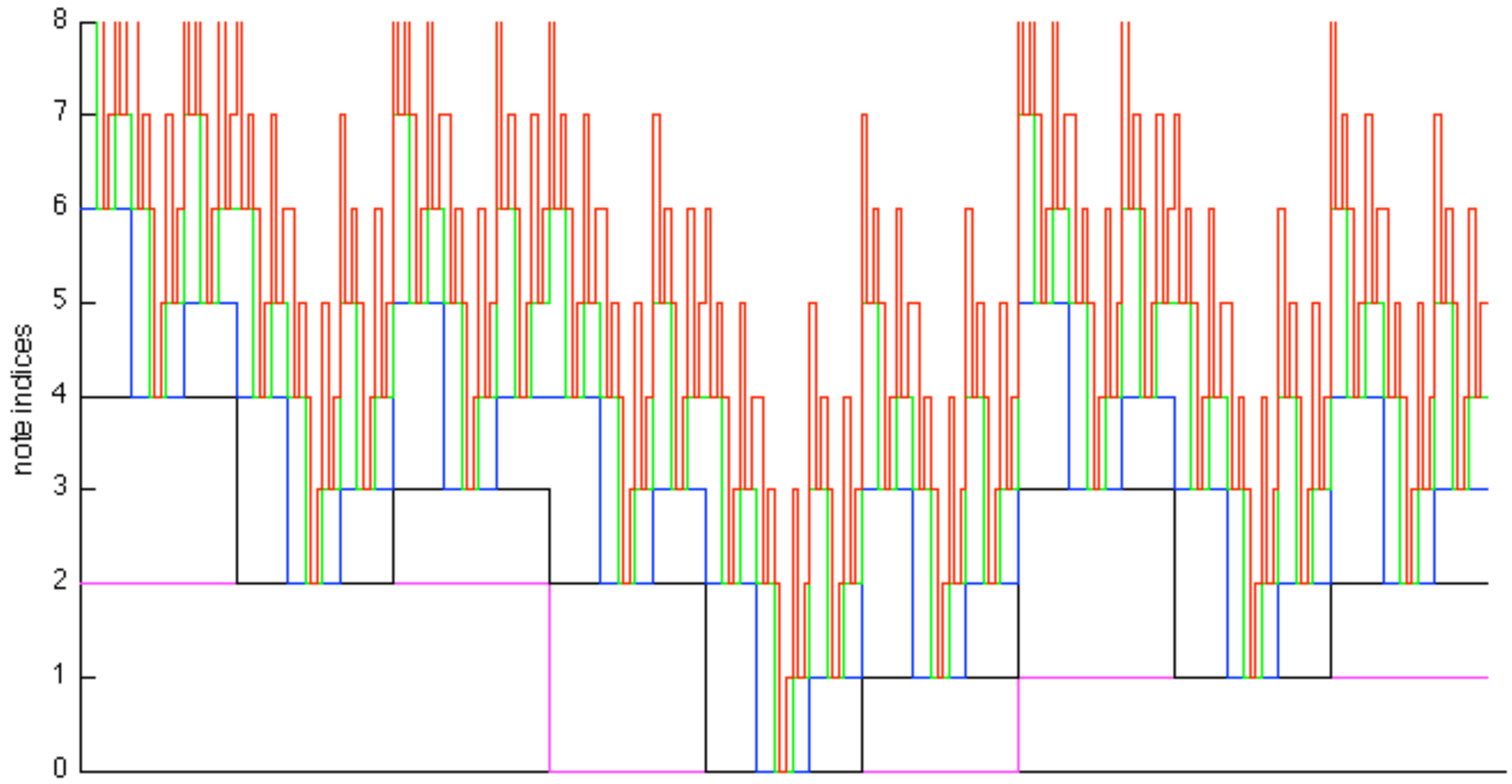




And Another...

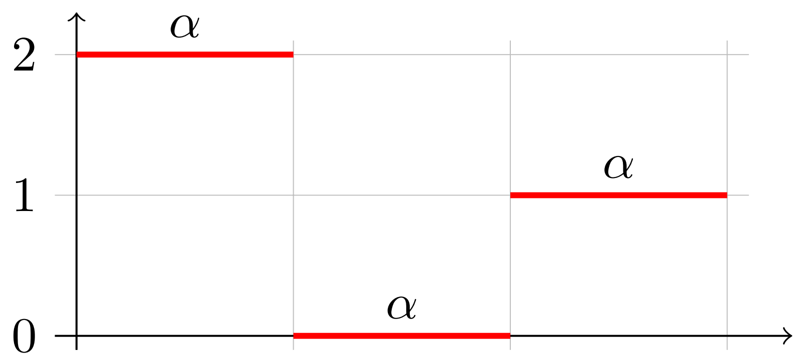
Pattern α

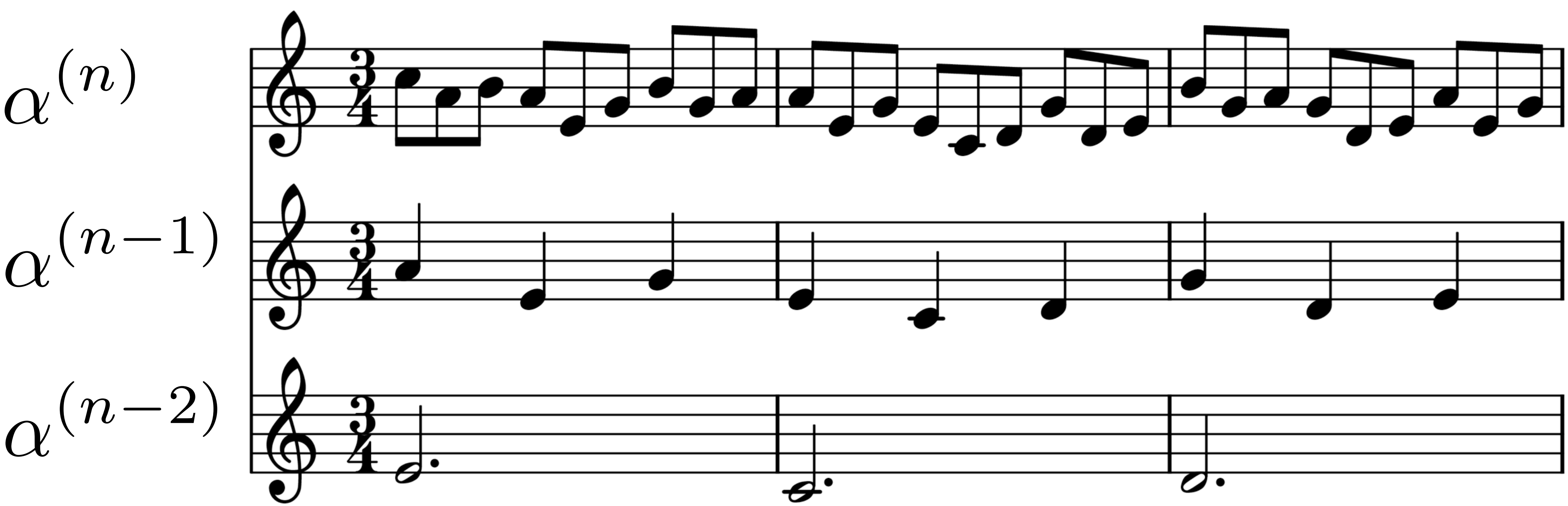




Self-Similarity

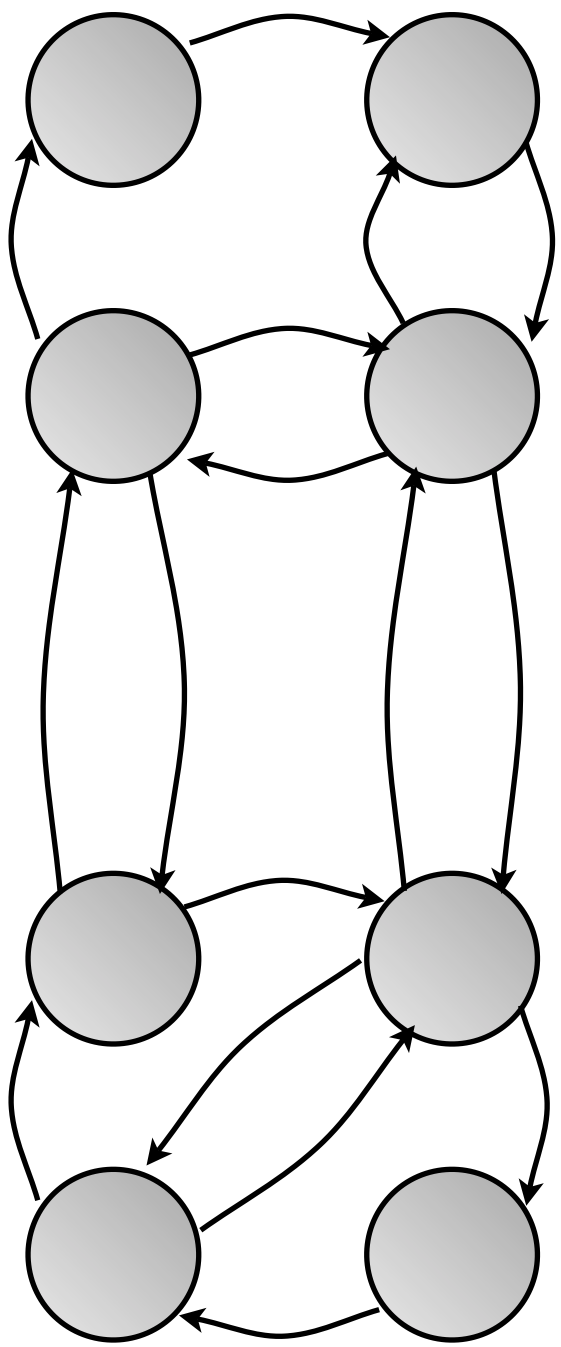
Pattern α



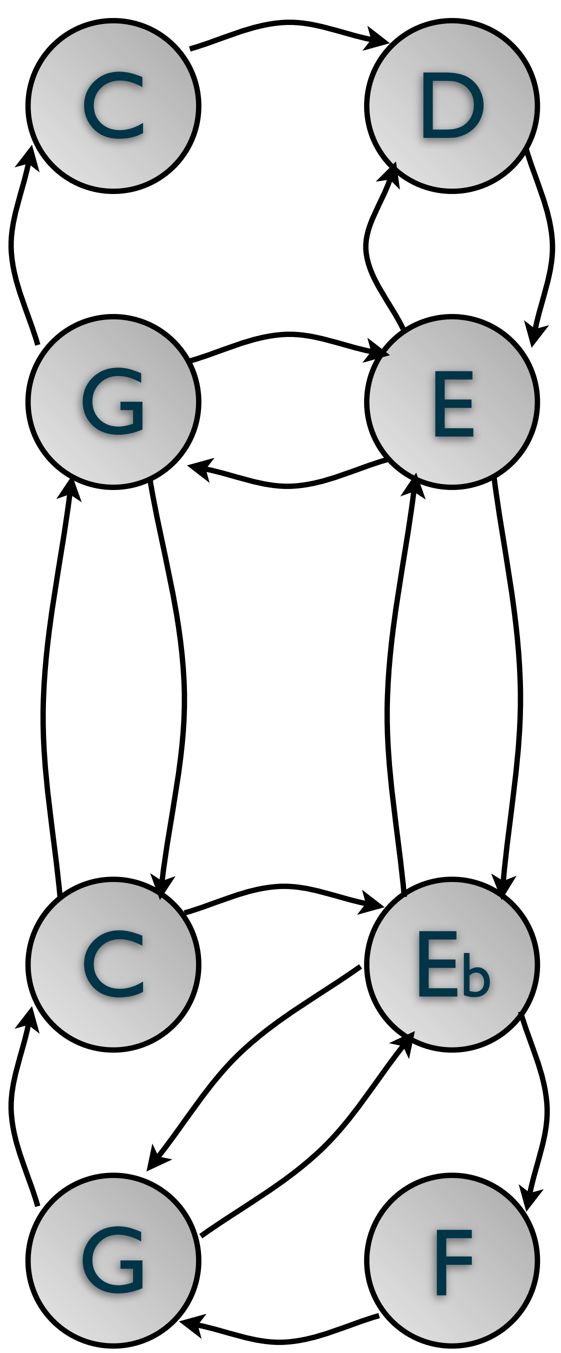


A Musical Markov Chain

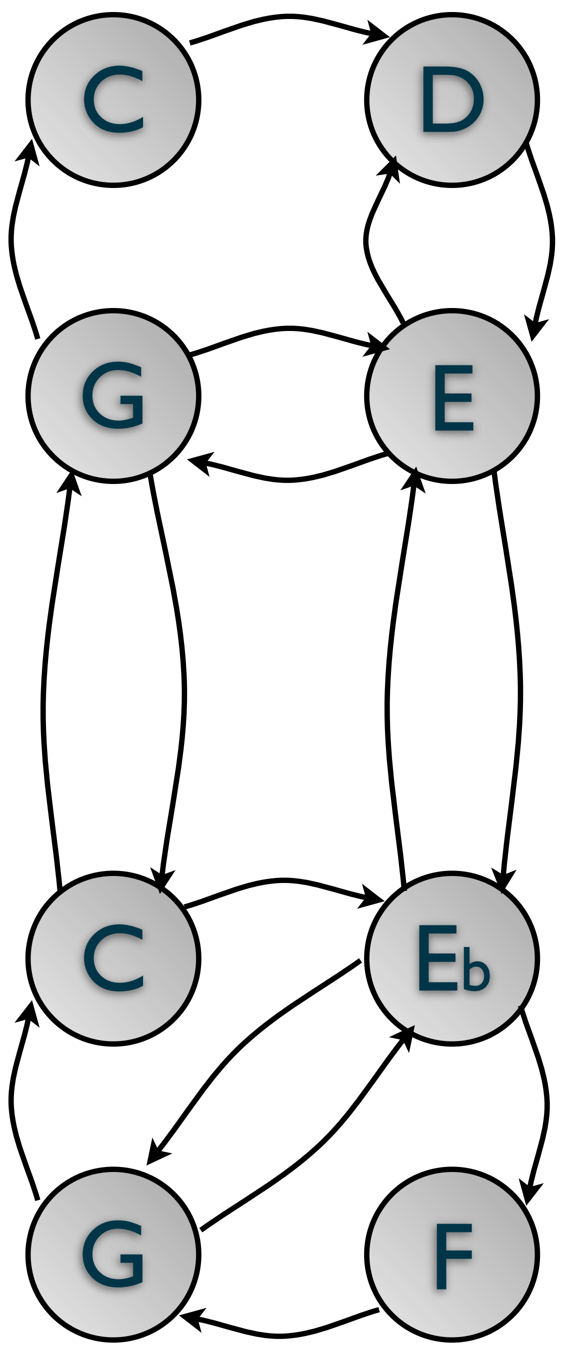
A Musical Markov Chain



A Musical Markov Chain



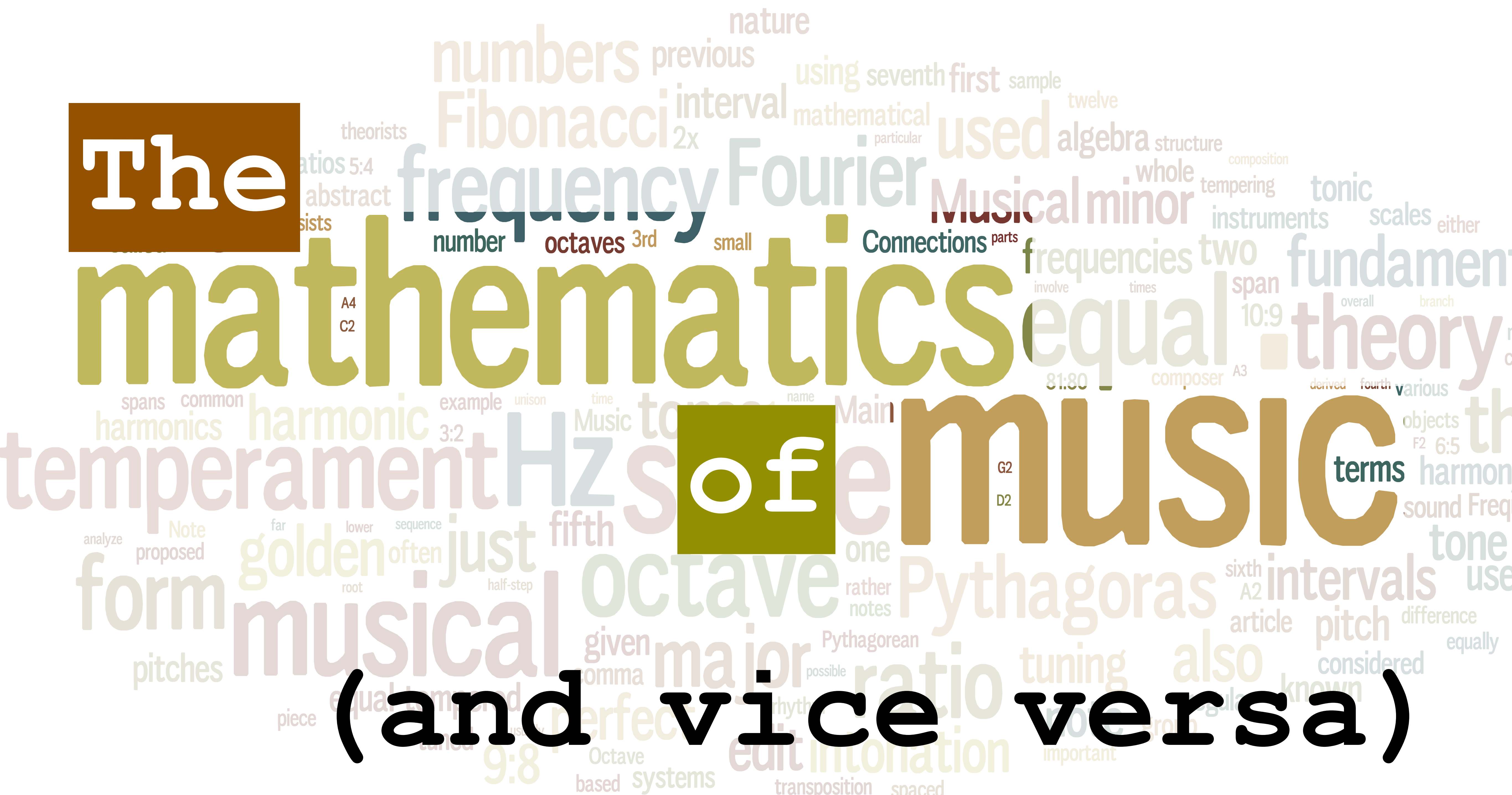
A Musical Markov Chain



A Musical Escher

A Musical Escher

Thanks!


The
mathematics
of
music
(and vice versa)
nature
numbers previous
using seventh first sample
twelve
theorists
Fibonacci interval
mathematical
2x particular
used algebra structure
composition
ratios 5:4
abstract
frequency Fourier
Musical minor
whole tempering
tonic
instruments
scales either
number octaves 3rd
small
Connections parts
frequencies two
fundament
involve times
span
overall branch
A4
C2
equal
10:9
theory
81:80
composer A3
derived fourth various
spans common
example unison
time
name
harmonics harmonic
3:2
Music
Main
objects
F2 6:5
temperament Hz
G2
D2
terms
harmon
sound Freq
analyze
Note far lower sequence
proposed
golden often just fifth
one
tone
root half-step
rather notes
Pythagoras
sixth intervals
A2
use
form
musical octave
article pitch difference
equally
given
comma
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possible
ratio tuning also
considered
known
pitches
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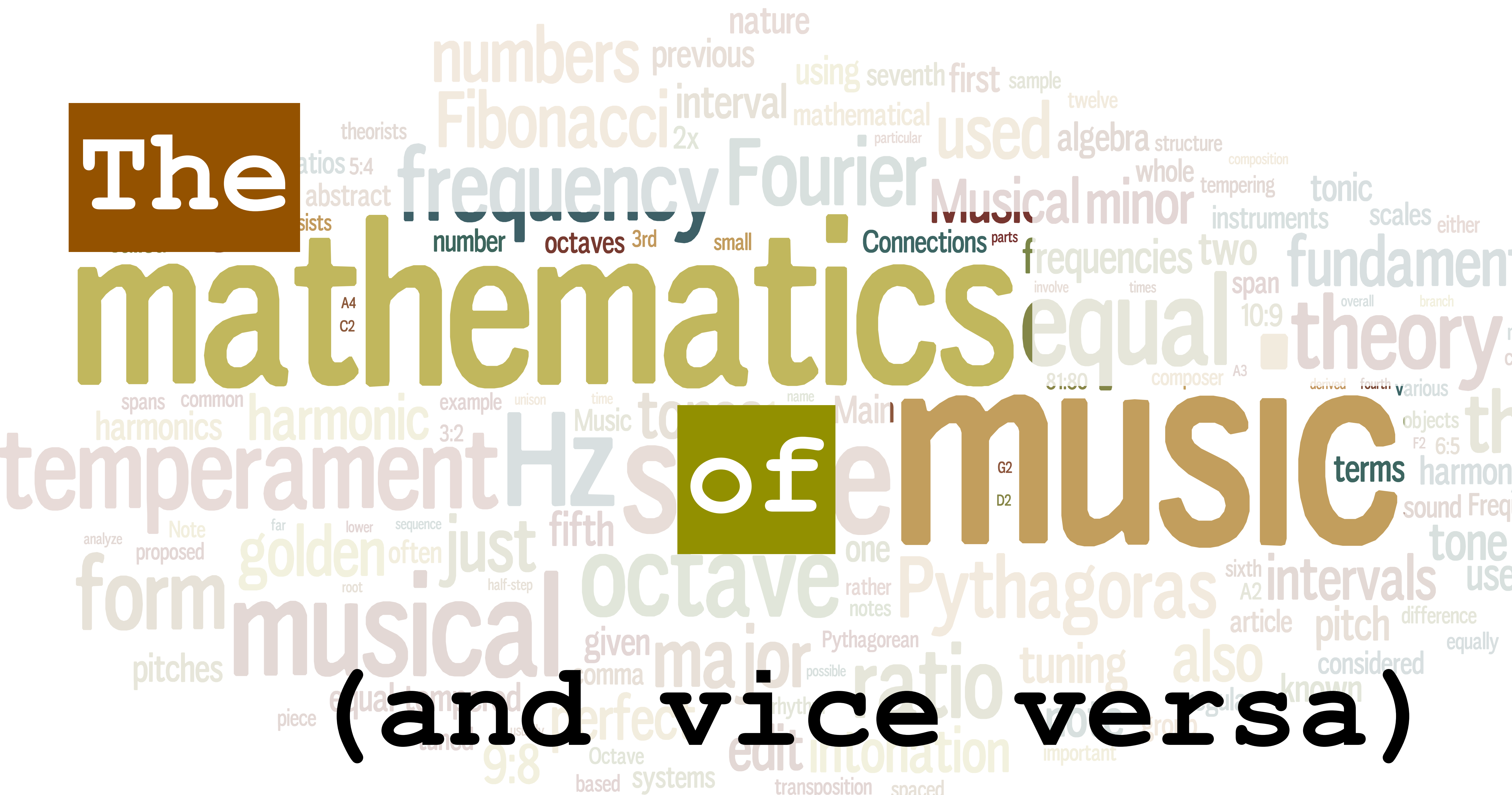
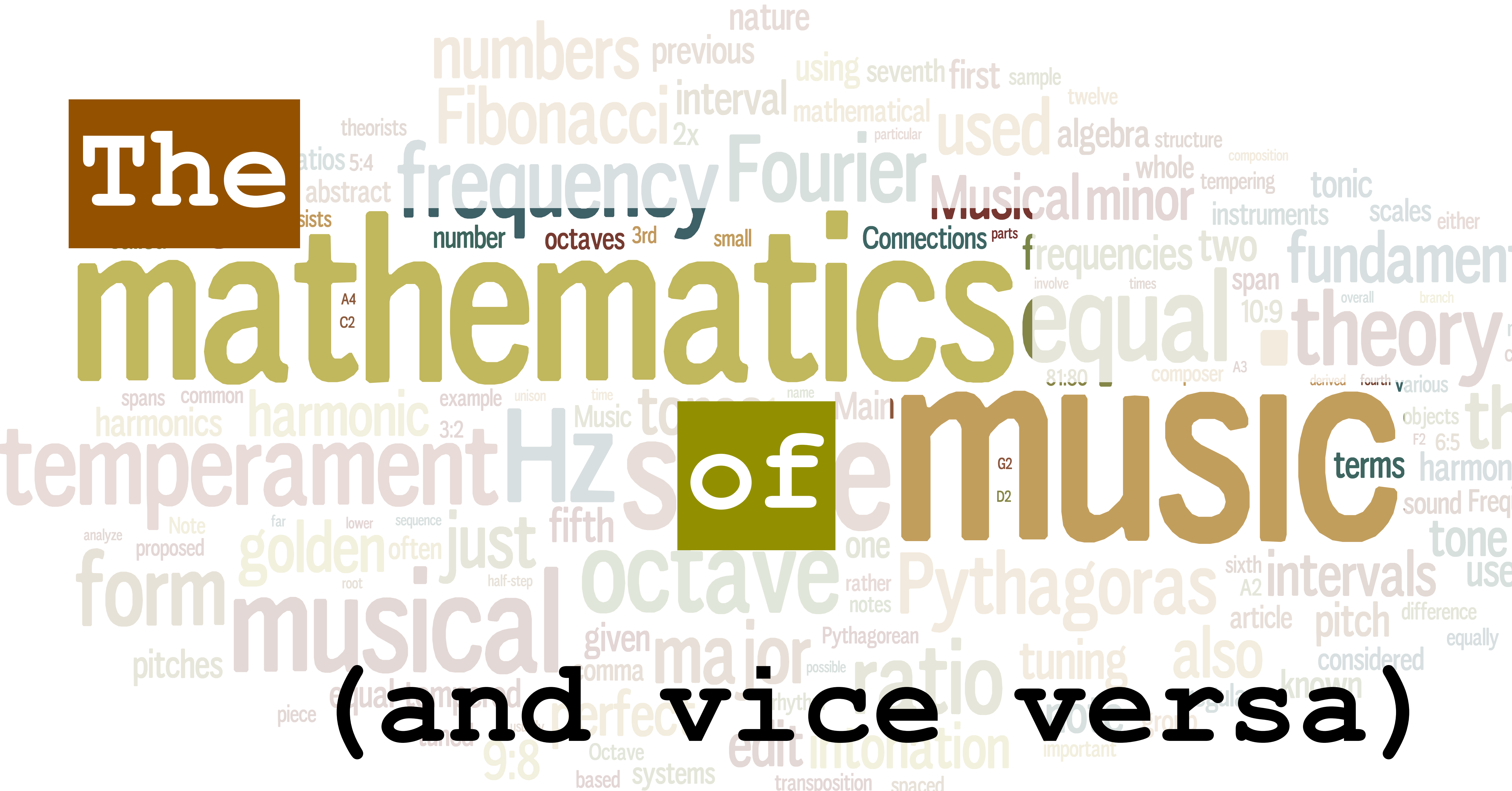
important
9:8
Octave
based systems
transposition spaced
intonation

Thanks!



mpendergrass@hsc.edu

Thanks!



mpendergrass@hsc.edu