# Mtg 15: 17 Feb

## Bayesian Binomial Estimation

↳ the type of problem we're doing

↳ coin with unknown head probability

↳ A priori beliefs about the coin are modeled by the <u>prior</u> <u>distribution</u> $\pi(p)$

i.e.

$$\pi(p) = P(P = p)$$ "prior for P"

↳ Observe $X = k$ heads out of $n$ flips

↳ What is the posterior distribution of $P$

i.e. $$\pi(p \mid k) = P(P = p \mid X = k)$$

By Bayes Theorem

$$\pi(p|k) = \frac{\pi(p)\,P(X=k|P=p)}{\int_0^1 \pi(p')\,P(X=k|P=p')\,dp'}$$

Note: denominator is a constant; in fact, the constant that makes $\int_0^1 \pi(p|k)\,dp = 1$

# Bayesian Shorthand

$$\pi(p \mid k) = \propto \pi(p)\, P(X=k \mid \pi=p)$$

"is proportional to"

A "nice" family of priors for Binomial Bayesian estimation:

<u>Beta Distributions</u>

$$\pi(p) = C \cdot p^{\alpha-1}(1-p)^{\beta-1}, \quad 0 \le p \le 1$$

and $$C = \frac{1}{\int_0^1 p^{\alpha-1}(1-p)^{\beta-1}\,dp} = \frac{1}{B(\alpha,\beta)}$$

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betaPDF.m | example.m

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close all;
clear all;
clc;

%% inputs

% alpha and beta params for beta distribution
a = 2;
b = 1;

%% computations

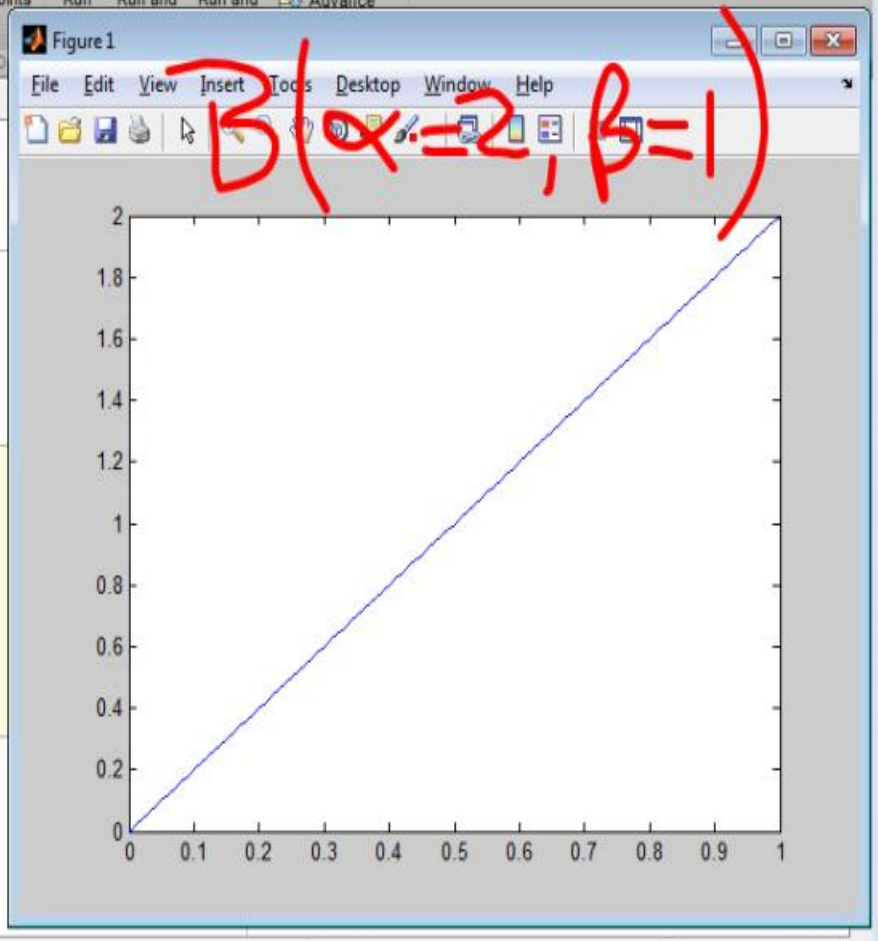
C = 1/beta(a, b);

p = (0 : 0.01 : 1);

% calculate the beta pdf
F = p.^(a-1) .* (1-p).^(b-1)*C;

%% output

plot(p, F);
```

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```
close all;
clear all;
clc;

%% inputs

% alpha and beta params for beta distribution
a = 1;
b = 2;

%% computations

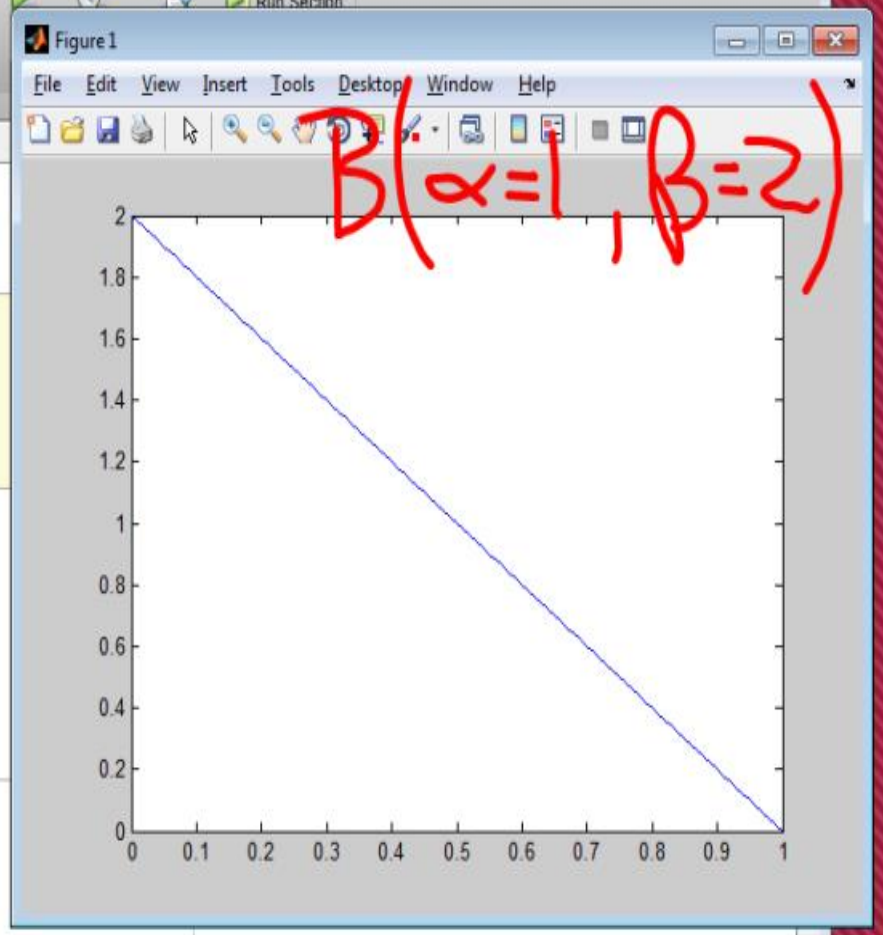
C = 1/beta(a, b);

p = (0 : 0.01 : 1);

% calculate the beta pdf
F = p.^(a-1) .* (1-p).^(b-1)*C;

%% output

plot(p, F);
```

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```
close all;
clear all;
clc;

%% inputs

% alpha and beta params for beta distribution
a = 1;
b = 1;

%% computations

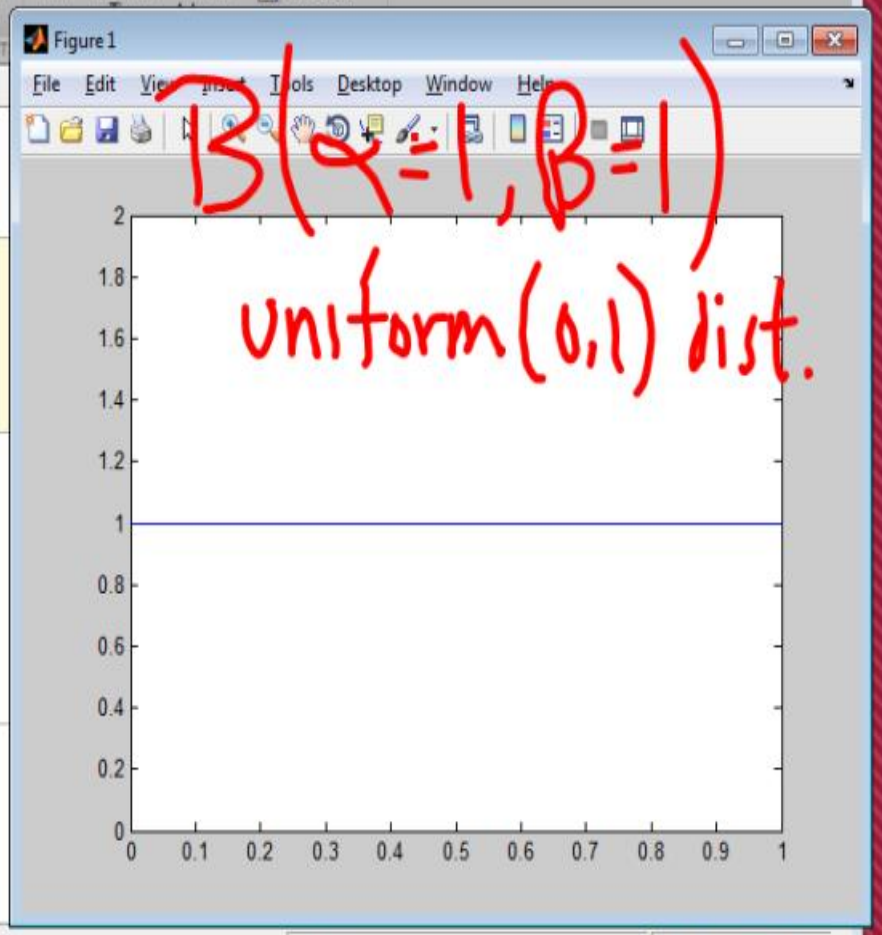
C = 1/beta(a, b);

p = (0 : 0.01 : 1);

% calculate the beta pdf
F = p.^(a-1) .* (1-p).^(b-1)*C;

%% output

plot(p, F);
```

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```
close all;
clear all;
clc;

%% inputs

% alpha and beta params for beta distribution
a = 2;
b = 2;

%% computations

C = 1/beta(a, b);

p = (0 : 0.01 : 1);

% calculate the beta pdf
F = p.^(a-1) .* (1-p).^(b-1)*C;

%% output

plot(p, F);
```

Figure 1

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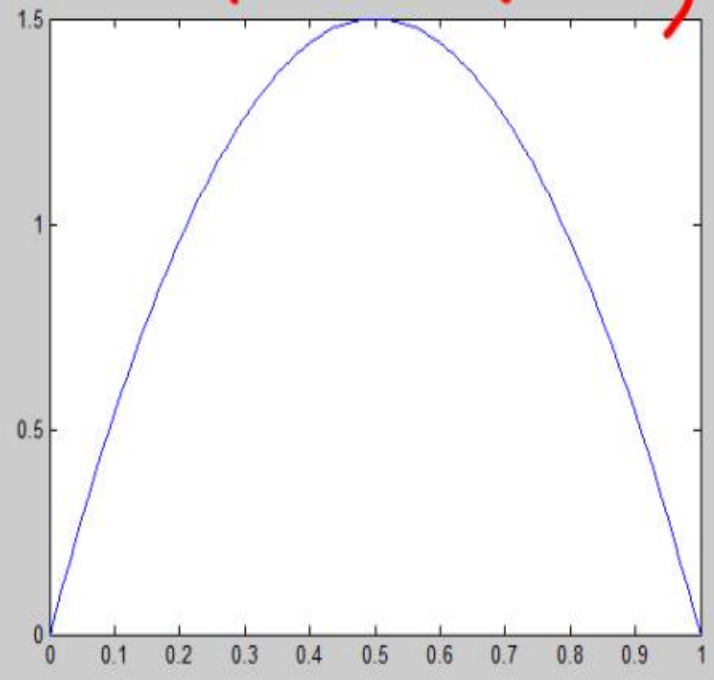
$B(\alpha=2, \beta=2)$

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```
close all;
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clc;

%% inputs

% alpha and beta params for beta distribution
a = 10;
b = 10;

%% computations

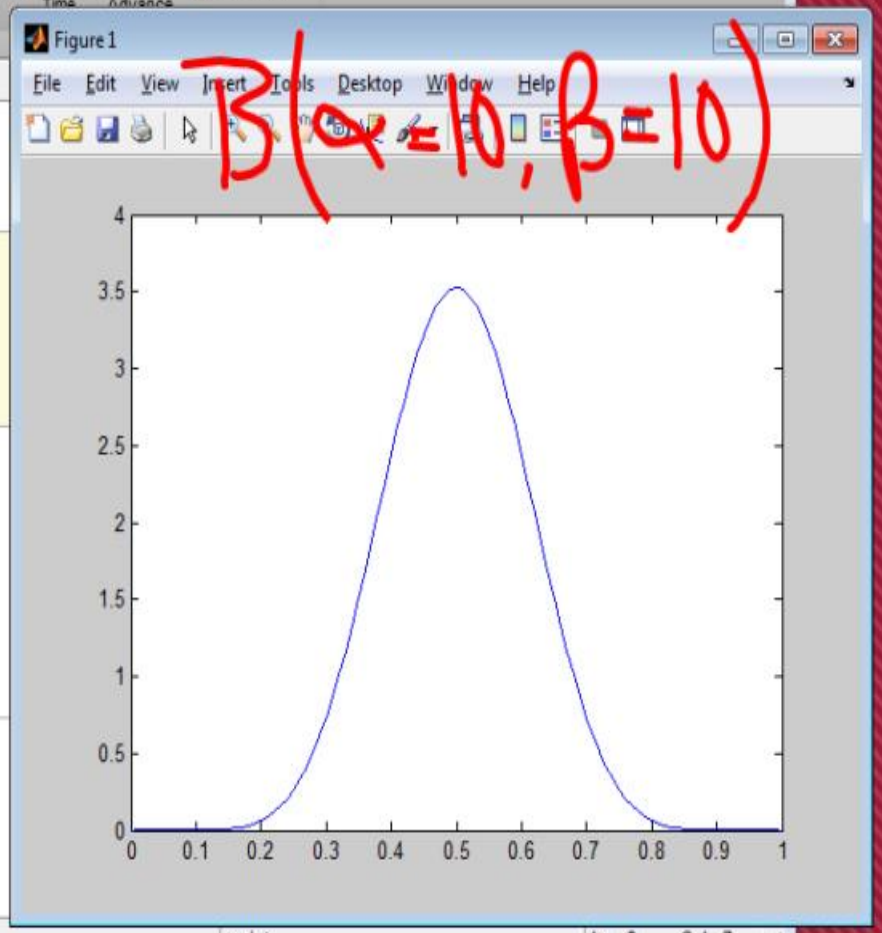
C = 1/beta(a, b);

p = (0 : 0.01 : 1);

% calculate the beta pdf
F = p.^(a-1) .* (1-p).^(b-1)*C;

%% output

plot(p, F);
```

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```
close all;
clear all;
clc;

%% inputs

% alpha and beta params for beta distribution
a = 10;
b = 20;

%% computations

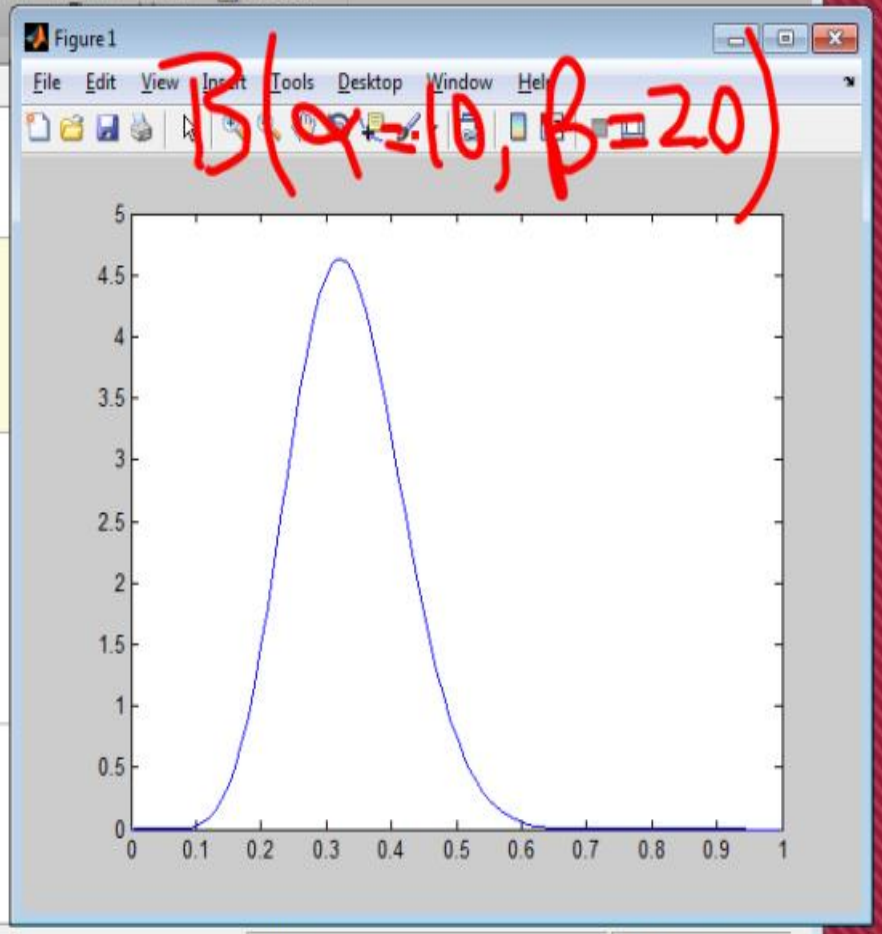
C = 1/beta(a, b);

p = (0 : 0.01 : 1);

% calculate the beta pdf
F = p.^(a-1) .* (1-p).^(b-1)*C;

%% output

plot(p, F);
```

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