On my honor, I have neither given nor received any aid on this work, nor am I aware of any breach of the Honor Code that I shall not immediately report.

Pledged: ______________________________

Print Name: ______________________________

*Solve the following problems. The solution to each problem should be clearly presented, in a logical and coherent fashion. Begin each solution on a new sheet of paper. You can use your class notes, my posted lecture notes, and the solutions to all previous assignments in working these problems. You can also use any standard calculus book. You may use Maple (or another CAS) only on problem 6. No other outside resources are allowed (including on-line resources). This exam is due Monday, November 12, at the beginning of class.*

1. (Envelopes) When a musician *accents* a note, he or she will typically make the first part of the note (the "attack") very forceful and loud, while the steady-state part of the note will be at a somewhat reduced volume. The "accent edge" in the figure below models this type of attack.

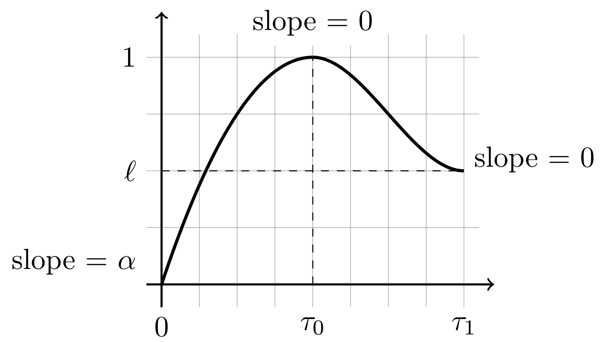


Figure 1. An "accent edge."

Here, $\alpha$ is the slope of the edge at time $t = 0$, $\tau_0$ is the time at which the accent edge is at a maximum, and $\tau_1$ is the end-time for the edge. Use the cubic rising edge model we derived in class to develop a model for the accent edge. Define the accent edge piecewise, using one cubic rising edge to model the first part of the accent edge (up to time $\tau_0$), and then a translated, scaled, and flipped cubic rising edge to model the second part of the accent edge (from time $\tau_0$ to time $\tau_1$). Give a general formula for the accent edge in terms of the rising edge. Give an explicit formula for the accent edge for the case $\tau_0 = 1$, $\alpha = 2$, $\tau_1 = 2$, and $\ell = 0.5$,

2. (Instantaneous Frequency) Recall that if $s(t) = \cos(2\pi g(t))$, then the *instantaneous frequency function* for $s$ is defined by $f_i(t) = g'(t)$.
   (a) Let $s(t) = \cos(2\pi f_0 t + \frac{W}{2} e^{-at} \sin(2\pi f_m t))$. Find the instantaneous frequency function for $s$.
   (b) Suppose $s(t) = \cos(2\pi g(t))$, and the instantaneous frequency function for $s$ is $f_i(t) = f_0 + \rho t^n$. Find an explicit formula for $s(t)$.

3. (Pitch Siren) Recall that *pitch* is the base-2 logarithm of frequency. Accordingly, the *instantaneous pitch* of $s(t) = \cos(2\pi g(t))$ is $p_i(t) = \log_2(f_i(t))$, where $f_i$ is the instantaneous frequency function for $s$. Suppose that the instantaneous pitch function is given by

$$p_i(t) = p_0 + \frac{W}{2} \sin(2\pi f_m t)$$

Find a formula for $s(t)$. (Your formula will contain an integral.)

4. (Power and Phase Spectra) Consider the waveform

$$s(t) = \sum_{k=1}^{4} \frac{1}{k^2} \cos(2\pi k f_0 t) + \frac{1}{k} \sin(2\pi k f_0 t).$$

Find the power spectrum of $s$, and the phase spectrum of $s$. Plot each as a stem plot.

5. (Parseval's Relation) Let $\alpha, \beta \in (-1, 1)$, let $T > 0$, and consider the waveforms $s$ and $w$ defined on $[0, T]$ by

$$s(t) = \sum_{k=1}^{\infty} \alpha^{k-1} \cos(2\pi k f_0 t) + \beta^{k-1} \sin(2\pi k f_0 t),$$

$$w(t) = \sum_{k=1}^{\infty} \beta^{k-1} \cos(2\pi k f_0 t) - \alpha^{k-1} \sin(2\pi k f_0 t),$$

where $f_0 = 1/T$.

(a) Find $\|s\|^2 = \int_0^T s^2(t)\, dt$.

(b) Find $\langle s, w \rangle = \int_0^T s(t) w(t)\, dt$.

6. (Fourier Transform) Consider the *exponentially enveloped sinsusoid*

$$s(t) = \begin{cases} e^{-at} \sin(2\pi f_0 t) & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

where $a > 0$ and $f_0 > 0$.

(a) Find the Fourier transform of $s$.

(b) Find the energy spectral density $E(f)$ for $s$.

(c) Plot both $s$ and the energy spectral density for the case $a = 1$, $f_0 = 440$.

(d) Describe how the energy spectral density changes as $a$ increases.