Applied Mathematics
Math 345

Project 2

October 12, 2012
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ON MY HONOR, I HAVE NEITHER GIVEN NOR RECEIVED ANY AID ON THIS WORK, NOR AM I AWARE OF ANY BREACH OF THE HONOR CODE THAT I SHALL NOT IMMEDIATELY REPORT.

Pledged: ____________________

Print Name: ____________________

Work together with your partner on the following problems. You may refer to your own notes, and to the *Introduction To Fourier Series* note sets for concepts, notation and terminology. This project is due Friday, October 19, at the beginning of class.

1. In this activity you will use the `trigSeries` function to verify some of the basic properties of Fourier series that we've discussed. (You can see an example of how to use this function in the file `trigSeriesExample.m`.) Your work for this problem should be saved in a script file called `project02_problem1.m`.

   (a) First, generate *random Fourier coefficients*, as follows:

   ```
   numCoefficients = 7;
   a = randn(1, numCoefficients);
   b = randn(1, numCoefficients);
   ```

   This produces two arrays Fourier coefficients: a (for the cosines) and b (for the sines). The entries in the arrays are normally distributed, with mean 0 and standard deviation 1. Now use these as inputs to the `trigSeries` function to produce a Fourier series. Use the following additional parameters:

   - fundamental frequency $f_0 = 440$.
   - duration $T = 3$;
   - sampling rate $f_s = 11025$.

   Call your function s. Create a plot consisting of four cycles of this function, with properly labelled axes.

   (b) Now compute the energy of s directly, as follows:

   ```
   % compute the time-spacing
   dt = 1/fs;
   % approximate the integral with a Riemann sum
   E1 = sum(s.^2)*dt;
   ```

   What integral are you approximating with this code?

   (c) Recall that Parseval's relation states that if

   $$s(t) = \sum_k a_k \cos(2\pi k f_0 t) + b_k \sin(2\pi k f_0 t)$$

   then

   $$\|s\|^2 = \frac{T}{2} \sum_k a_k^2 + b_k^2$$

   You can do this computation in MATLAB as follows:

   ```
   E2 = T/2 * sum(a.^2 + b.^2);
   ```

   Do this, and verify that you get the same result that you got in the previous question.

   (d) Parseval's relation actually goes further than this. If

   $$s(t) = \sum_k a_k \cos(2\pi k f_0 t) + b_k \sin(2\pi k f_0 t)$$

   and

   $$r(t) = \sum_k c_k \cos(2\pi k f_0 t) + d_k \sin(2\pi k f_0 t)$$

then a more general Parseval theorem says that

$$\langle s, r \rangle = \frac{T}{2} \left(\langle a, c \rangle + \langle b, d \rangle \right)$$

Note that the inner product on the left is an integral of two functions, while the inner products on the right are between vectors in $\mathbb{R}^m$. Verify this more general Parseval relation by generating a new random Fourier series r (use $n = 7$ again, and call your coefficient arrays c and d), and computing $\langle s, r \rangle$ directly and using the above equation.

2. The file `flute.E4.mat` contains a recording of a flute playing middle E (the E below middle A). The variable `s1` contains the recorded data, while `fs` and `n` contain the sampling rate and number of samples respectively. Middle E has a frequency of 330 Hz. Use a Fourier series with $n = 5$ to approximate this waveform. Compare the sound of your approximation to the original, and comment. Save your work in a MATLAB script called `project02_problem2.m`. (HINT: What is your T? your f_0? Now instead of calculating the first five harmonics of f_0, calculate the Fourier coefficients for the first five harmonics of $f = 330$ Hz. Use a Riemann sum to compute the integral. Once you've calculated the a_k's and b_k's, use the `trigSeries` function to construct your approximation.)

3. Repeat problem 2 for the oboe sound contained in the file `oboe.E4.mat`. Compare the sound of your approximation to the original, and comment. Save your work in a MATLAB script called `project02_problem3.m`.