

ON MY HONOR, I HAVE NEITHER GIVEN NOR RECEIVED ANY AID ON THIS WORK, NOR AM I AWARE OF ANY BREACH OF THE HONOR CODE THAT I SHALL NOT IMMEDIATELY REPORT.

Pledged: _____

Print Name: _____

Do these problems in your homework notebook, according to the same guidelines used for homework. You may use your class notes, and the lecture slides and notes posted on our class web-page. No other resources are allowed. This test is due at the beginning of class on Monday, October 14, 2013.

1. Events A and B in a probability space have the following probabilities:

$$P(A) = 0.60, \quad P(B) = 0.55, \quad P(A \cap B^c) = 0.30$$

- Find $P(A \cup B)$.
 - Find $P(B|A)$.
 - Find $P(A^c \cup B^c)$.
 - Find $P(A^c \cap B^c)$.
 - Are A and B independent?
2. An all-seeing, all-knowing genie informs you that the Washington Nationals baseball team will go 33-48 at home next season (i.e. 33 wins, 48 losses for games played at their home stadium). If you go to 19 home games next year chosen at random, what is the probability that the Nationals lose each one?
3. A seven-card hand is dealt from a well-shuffled deck of 52 ordinary playing cards. Find the probabilities of the following events:
- P (four of a kind, and three of another kind).
 - P (three of a kind, and two pairs).
 - P (at least one ace).
 - P (straight). (Note: a straight is 7 cards in sequence. An ace can be either high or low in a straight.)
4. In medical testing, a *false positive* occurs when the test indicates that the disease is present, when in fact no disease is present. A *false negative* occurs when the test indicates that no disease is present, when in fact disease is present. Let D be the event that disease is present, $+$ be the event that the test is positive, and $-$ be the event that the test is negative. Suppose that a certain test has false positive rate $P(+|D^c) = 0.001$, and false negative rate $P(-|D) = 0.0005$. Suppose also that the prevalence of the disease is relatively rare: $P(D) = 10^{-6}$.
- If the test comes back positive, what is the probability that the patient has the disease?
 - If the test comes back negative, what is the probability that the patient does not have the disease?
5. A bag of candy contains 6 Reese's Cups, and 9 Snickers bars. Roll a fair six-sided die, and if the outcome is k , draw k pieces of candy out of the bag.
- Find the probability that you get at least one Reese's Cup.
 - Let Y be the number of Reese's Cups you get. Find the probability mass function for Y .
6. Urn I contains a one-dollar bill, Urn II contains two one-dollar bills and a ten-dollar bill, and Urn III contains three one-dollar bills, two ten-dollar bills, and a hundred-dollar bill. But the urns aren't labeled, so you don't know which is which. Choose an urn at random, and then pick a bill at random from that urn. Let X be the value of the bill you get.
- Find the probability mass function for X .

- (b) Suppose that $X = 1$. What is the probability that the bill came from Urn I?
7. Compute the probability that a 5-card poker hand is void in at least one suit.
8. If there are 12 strangers in a room, what is the probability that no two of them celebrate their birthday in the same month? Clearly state the basic assumptions you are making.
9. Reorder the digits 1 through 9 randomly. Call a reordering *deranged* if digit d does not appear in place d , for all $d = 1, 2, \dots, 9$. (So for instance, 825146973 is not deranged, since the digit 2 appears in place 2; but the reordering 715832946 is deranged.) Find the probability that a reordering is deranged.
10. A man has n keys, one of which will open his door.
- (a) If he tries the keys at random, discarding those that do not work, what is the probability that he will open the door on his k^{th} try?
- (b) What is the probability if he does not discard previously tried keys?