On my honor, I have neither given nor received any aid on this work, nor am I aware of any breach of the Honor Code that I shall not immediately report.

Pledged: ______________________________

Print Name: ______________________________

Do these problems in your homework notebook, according to the same guidelines used for homework. You may use your class notes, and the lecture slides and notes posted on our class web-page. No other resources are allowed. This test is due at 3 PM on Tuesday, November 26, 2013.

1. Pick a point at random on a rod of length L, and break the rod into two pieces at this point. What is the probability that the larger piece is more than n times as long as the smaller piece? (Assume $n > 1$.)

2. Consider a random variable X with the probability density function

$$p(x) = \begin{cases} C(2-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   Here, C is a constant.

   (a) Find the value of C that makes p a valid probability density function.

   (b) Find the cumulative distribution function for this distribution.

   (c) If m is a number such that $P(X \le m) = 1/2$, we say m is a *median* of the distribution of X. Find the median of this distribution.

3. Consider the function

$$F(x) = \frac{1}{1+e^{-ax}}, \quad -\infty < x < \infty$$

   where $a > 0$ is a constant.

   (a) Prove that F is a cumulative distribution function.

   (b) Find the probability density function for F.

   (c) Prove that the probability density function is symmetric about the line $x = 0$.

4. Let X have the probability density function

$$p(x) = \begin{cases} xe^{-x} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the moment generating function for X.

   (b) Find $E\left[X^k\right]$ for $k \in \mathbb{N}$.

5. Let X be uniformly distributed on $[0, \pi]$, and let $Y = \sin(X)$.

   (a) Find the expected value of Y.

   (b) Find the probability density function for Y.

6. Let X and Y be independent random variables, both uniformly distributed on $[0, 1]$.

   (a) Find the probability density function for $Z = \max(X, Y)$.

   (b) Find the probability density function for $W = \min(X, Y)$.

7. Let X be exponentially distributed with parameter $\lambda > 0$. Show that $P(X > t + s | X > s) = P(X > t)$ for all $s, t \ge 0$.